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Effect of Omega(782) resonance on the response functions for d(e,e′ π^+)nn reaction

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In the present work we study the effect of the Omega ($\omega(782)$) resonance on the response functions for the incoherent positive pion electroproduction on the deuteron at different values of the squaredfour-momentum transfers (Q^2) and the virtual photon lab energy (k_0^{lab}). The study is carried out in the impulse approximation (IA), i.e. the final state interactions are neglected. The elementary amplitude for pion electroproduction is taken from the MAID-2007 model. The effect seems to be very small and slightly increases with increasing virtual photon lab energy.

Keywords: pion electroproduction; the $\omega(782)$ resonance; invariant amplitudes; CGLN amplitudes; structure functions; the impulse approximation (IA).

Introduction

The pion is now known to exist in three charge states, π^+ , π^- and π^0 , with masses of 139.6 MeV for charged pions and 135.0 MeV for the neutral pion. The Pion photo and electroproduction is presently one of the main sources of our information on the structure of nucleons. With the advent of a new generation of high duty-factor electron accelerators as MAMI (Mainz), ELSA (Bonn), and Jefferson Lab (Newport News) as well as modern laser back scattering facilities as LEGS (Brookhaven) and GRAAL (Grenoble), the photo- and electroproduction of pions on a proton have been studied thoroughly both theoretically and experimentally.

The elementary amplitude of pion photo- and electroproduction on free nucleons is one of the main components of the analysis of these reactions for nuclei. In order to study pion electroproduction on complex nuclei, it is first necessary to understand the production process on the nucleon. To this end there have been extensive studies, for example, in [1].

The basic interaction in meson elctoroproduction is as follows: a virtual photon is incident on a target nucleus and interacts with its constituents. As a result, a pseudoscalar meson is produced along with other particles. Two kinds of processes depending on the nature of the other particles produced in this interaction are found: coherent and incoherent processes. In the coherent process [2-3], the meson is produced with the target nucleus maintaining its initial character. Thus, the interaction starts with a virtual photon and a nucleus, and ends up with a meson and the same nucleus, i.e γ^* XA $--\ \rightarrow\ \pi$ NXA-1, where A is the mass number of the target nucleus. The process is labeled "coherent" because all nucleons in the nucleus participate in the process coherently, leading to a coherent sum of the individual nucleon contributions.

In the incoherent process[4], the nucleus ruptures and thus fails to maintain its initial identity. The meson is produced in association with a nucleon (or an excited state of the nucleon) and a new recoil "daughter" hadronic system. Thus, the interaction starts with a virtual photon and a nucleus and ends up with a meson, a free nucleon (or its excited state) and a new hadronic system, i.e. γ^* XA $--\ \rightarrow\ \pi$ NXA-1. The process is labeled as "incoherent" because it occurs in kinematic and physical conditions similar to those that occur in the process where a meson is produced from a free nucleon.

Pion electroproduction on the deuteron near the threshold has been studied in the impulse approximation using an approach based on the unitary transformation method both experimentally [5-6] and theoretically [7-9].

Nucleon resonances are excited states of nucleon particles, often corresponding to one of the quarks having a flipped spin state, or with different orbital angular momentum when the particle decays. The symbol format is given as N (M)L2I 2J, where M is the particle's approximate mass, L is the orbital angular momentum of the nucleon-meson pair produced when it decays, and I and J are the particle's isospin and total angular momentum, respectively. Since nucleons are defined as having 1/2 isospin, the first number will always be 1, and the second number will always be odd. When discussing nucleon resonances, sometimes the N is omitted and the order is reversed, giving L2I 2J(M). For example, a proton can be symbolized as "N (939) S11" or "S11 (939)", delta resonances can be symbolized as "D 33(1232)" and omega can be symbolized as " $\omega(782)$ "

In this paper the effect of $\omega(782)$ on the semi-exclusive structure functions of the incoherent π^+ meson electroproduction off the deuteron is studied at 0.01, 0.05 and 0.1 GeV^2 four momentum transfer (Q^2) and different values of the incident virtual photon lab energy k_0^{lab}. The present paper is organized as follows; the formalism of π^+ electroproduction off the deuteron in the IA is briefly given in section 2. The results are summarized and some discussion is presented in section 3. At the end the summary and an outlook are presented.

Formalism

The basic formalism for electromagnetic single pion production on the deuteron has been presented in detail for the case of electroproduction [10]. Therefore, we review here only the most important ingredients with due extensions to electroproduction according to the additional contributions from charge and longitudinal current components.

Kinematics

The kinematics of the charged pion electroproduction in the one-photon exchange approximation is very similar to photoproduction in replacing the real photon by a virtual one with longitudinal and transverse momenta[11]:

$$\gamma^*(k) + d(p_d) \longrightarrow (p_1) + n(p_2) + \pi^+(q), \tag{1}$$

where $k = (k_0, \vec{k}), p_d = (E_d, \vec{d}), q = (\omega, \vec{q}), p_1 = (E_1, \vec{p_1}), p_2 = (E_2, \vec{p_2})$ denote the four-momenta of the incoming virtual photon,initial deuteron, the outgoing pion and the two outgoing nucleons, respectively. The energies are given by:

$$E_d = \sqrt{M_d + \vec{d}}, \;\; E_1 = \sqrt{M + \vec{p_1}}, \;\; E_2 = \sqrt{M + \vec{p_2}}, \;\; \omega = \sqrt{m_\pi + \vec{q}}. \tag{2}$$

As coordinate system we choose a right-handed orientation with z-axis along the photon momentum $\vec{q}$ and y-axis perpendicular to the scattering plane along $\vec{k_e} \times \vec{k_e}$. We distinguish in general three planes: (i) the scattering plane spanned by the incoming and scattered electron momenta, (ii) the pion plane, spanned by the photon and pion momenta, which intersects the scattering plane along the z-axis with an angle ϕ_π , and (iii) the nucleon plane spanned by the momenta of the two outgoing nucleons intersecting the pion plane along the total momentum of the two nucleons. This is illustrated in Figure 1.

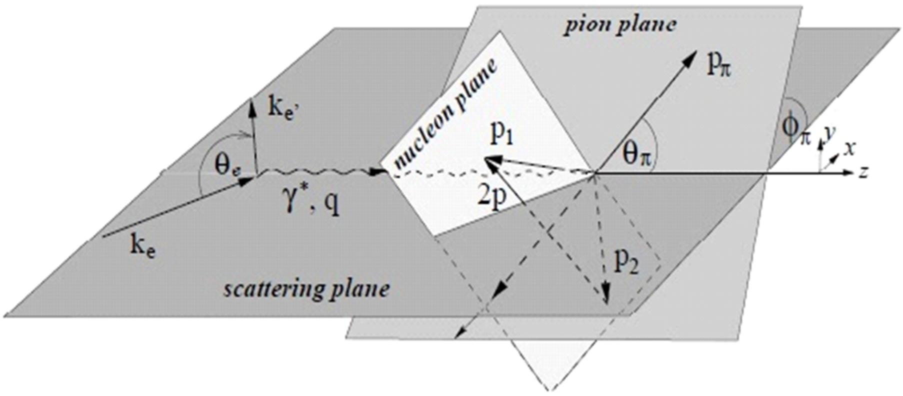

Figure 1. Kinematics of incoherent single pion electroproduction from the deuteron.

The T-matrix

All observables are determined by the T-matrix elements of the electromagnetic pion production current $J\gamma\pi$ between the initial deuteron and the final πNN states:

$$T_{sm_sum_d} = -^{(-)} \left\langle \vec{p_1}\vec{p_2}sm_s, \vec{p_\pi} | J_{\gamma\pi,\mu^{(0)}} | \vec{p_d} 1 m_d \right\rangle, \tag{3}$$

where s and m_s denote the total spin and its projection on the relative momentum of the outgoing two nucleons, and m_d correspondingly the deuteron spin projection on the z-axis as quantization axis.

Introducing a partial wave decomposition of the final states, one finds:

$$T_{sm_sum_d}(W, Q^2, p_\pi, \Omega_\pi, \Omega_p) = e^{i(\mu + m_d - m_s)\varphi_{pn}} t_{sm_sum_d}(W, Q^2, p_\pi, \theta_\pi, \theta_p, \varphi_{p,\pi}), \tag{4}$$

where the small t-matrix depends besides W, Q^2 and $p\pi$ only on θ_π, θ_p and the relative azimuthal angle $\phi_{p\pi} = \phi_p - \phi_\pi$. We had shown in [4] that, if parity is conserved, the following symmetry relation holds for $\mu = \pm 1$:

$$t_{s-m-_su-m_d}(W, Q^2, p_\pi, \theta_\pi, \theta_p, \varphi_{p,\pi}) =$$

$$= (-)^{s+\mu+m_d+m_s} t_{sm_s u m_d}(W, Q^2, p_\pi, \theta_\pi, \theta_p, -\varphi_{p,\pi}). \tag{5}$$

In the present work we include as e.m. current the elementary one-body pion production current of MAID-2007 [12] which has been developed for nuclear applications for photon energies up to 2 GeV. It contains Born terms, nucleon resonances P33(1232), P11(1440), D13(1520), S11(1535), S31(1620), S11(1650), D15(1675), F15(1680), D33(1700), P13(1720), F35(1905), P31(1910), F37(1950) and vector meson exchange, see Figure 2.

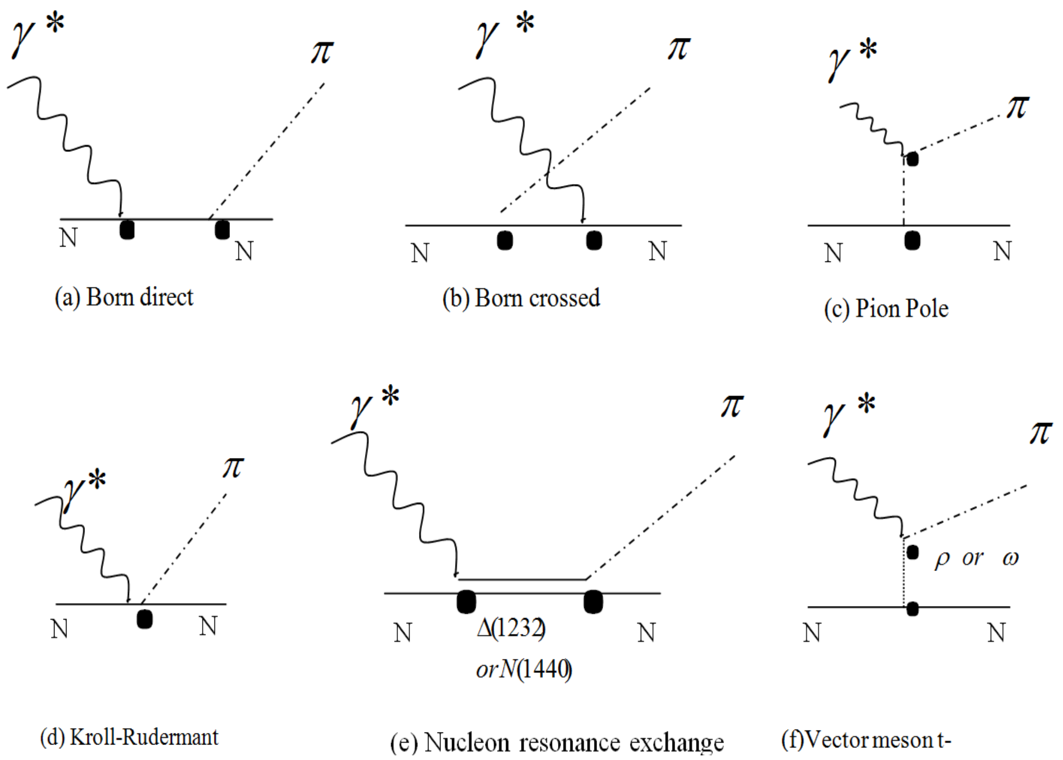

Figure 2. Diagrammatic representation of the elementary pion electroproduction on the nucleon. Born terms: (a)-(d) nucleon, crossed nucleon, pion poles and Kroll-Rudermann contact term; (e): resonance term; (f): vectormeson exchange.

For the IA contribution, where the final state is described by a plane wave, antisymmetrized with respect to the two outgoing nucleons [13]:

$$T^{IA}_{sm_s u m_d} = \langle \vec{p} s m_s, \vec{p_\pi} | \left[j_{\gamma\pi,\mu}(1) + j_{\gamma\pi,\mu}(2) \right] | 1 m_d \rangle$$

$$= \sqrt{2} \sum_{m_s'} \left(\left\langle s m_s | \left\langle \vec{p_1} | j_{\gamma\pi,\mu}(W_{\gamma N_1}, Q^2) | \times \vec{p_d} - \vec{p_2} \right\rangle \phi_{m_s' m_d}(\frac{1}{2}\vec{p_d} - \vec{p_2}) | 1 m_s' \right\rangle - (1 \leftrightarrow 2) \right) \tag{6}$$

where $j_{\gamma\pi,\mu}$ denotes the elementary pion photoproduction operator of the MAID-2007 model, $W_{\gamma N1}$ the invariant energy of the $\gamma N1$ system, $\vec{p_{1/2}} = (\vec{q} + \vec{p_d} - \vec{p_\pi})/2 \pm \vec{p}$.

Furthermore, $\phi_{m_s m_d}\vec{p}$ is related to the internal deuteron wave function in momentum space by:

$$\langle \vec{p}, 1 m_s | 1 m_s \rangle^{(d)} = \phi_{m_s m_d}\vec{p} = \sum_{L=0,2} \sum_{m_l} i^L (L m_L 1 m_s | 1 m_d) u_L(p) Y_L m_L(p). \tag{7}$$

Cross section and structure functions

The well-known spectator model, in which the pion is produced on a single nucleon inside the deuteron whereas the other nucleon acts as a pure spectator, is used to produce the matrix element of electroproduction on the deuteron.

The final expression for the semi-exclusive differential cross section is defined in [11], and the reader is referred to this work of the following expressions.

$$\frac{d^3\sigma}{dE_{e'}d\Omega_{e'}d\Omega_{\pi}^{c.m.}} = \frac{\alpha}{(2\pi)^2(Q^2)^2}\frac{p_{e'}}{p_e}$$

$$\left(\bar{\rho}_L R_L + \bar{\rho}_T R_T + \frac{1}{\sqrt{2}}\bar{\rho}_{LT} R_{LT} cos\varphi_{\pi}^{c.m.} - \bar{\rho}_{TT} R_{TT} cos2\varphi_{\pi}^{c.m.}\right) \tag{8}$$

where the structure functions $R_\alpha(\alpha = L, T, LT, TT)$ are given in detail by

$$R_L = W_{00},\ \ R_T = W_{11},\ \ R_{LT} = -\sqrt{2}\Re e W_{10},\ \ R_{TT} = W_{1-1}. \tag{9}$$

These structure functions depend on the invariant mass, the squared fourmomentum transfer Q^2, and on the pion angle $\theta_{\pi}^{c.m.}$

Results and Discussion

In this section we present and discuss our numerical results for the structure functions of positive and negative pion electroproduction from the deuteron in the IA. As already mentioned, the realistic MAID-2007 model [12] has been used for the evaluation of the elementary pion electroproduction operator on the free nucleon.

The electromagnetic production amplitude is parameterized in terms of CGLN amplitudes given as numerical tables in the pion-nucleon c.m. frame. This amplitude had to be generalized to an arbitrary frame of reference in order to be incorporated into the reaction on the deuteron. This was achieved by constructing Lorantz invariant amplitudesfrom the MAID-2007 model. This generalized elementary production operator was then used to evaluate pion electroproduction off the deuteron.The numerical evaluation is based on theGauss integration for calculation of the matrix element of the MAID operator using for the deuteron wave function an analytical parameterization of the S - and D-waves of the Bonn potential in the momentum space [14].

Figures 3-11 present the angular distribution for the four structure functions at different values of the four momentum transfer Q^2 and the virtual photon lab energy k_0^{lab}. The dotted lines indicate the situation when the $\Omega(782)$ resonance contributions are eliminated from the elementary process and the solid lines show the case when this contributions are added. It is clear that larger contributions come from the longitudinal (R_L) and transverse (R_T) structure functions.

In Figure 3, where the four momentum transfer $Q^2 = 0.01$ GeV^2 and the virtual photon lab energy $k_0^{lab} = 300$ MeV, the contributions of $\Omega(782)$ are not noticeable for all structure functions R_L, R_T, R_{TT} and R_{LT}.

An increase in the virtual photon lab energy to $k_0^{lab} = 350$ MeV at the same fourmomentum transfer $Q^2 = 0.01$ GeV^2 (Figure 4) showed that the contributions of $\Omega(782)$ were still very small for R_L, for R_T a small increase at the forward angles and a small decrease at the backward angles was noticed, for R_{LT} a small decrease at the forward and backward angles was observed, and for R_{TT} a small effect was found near the peak region.

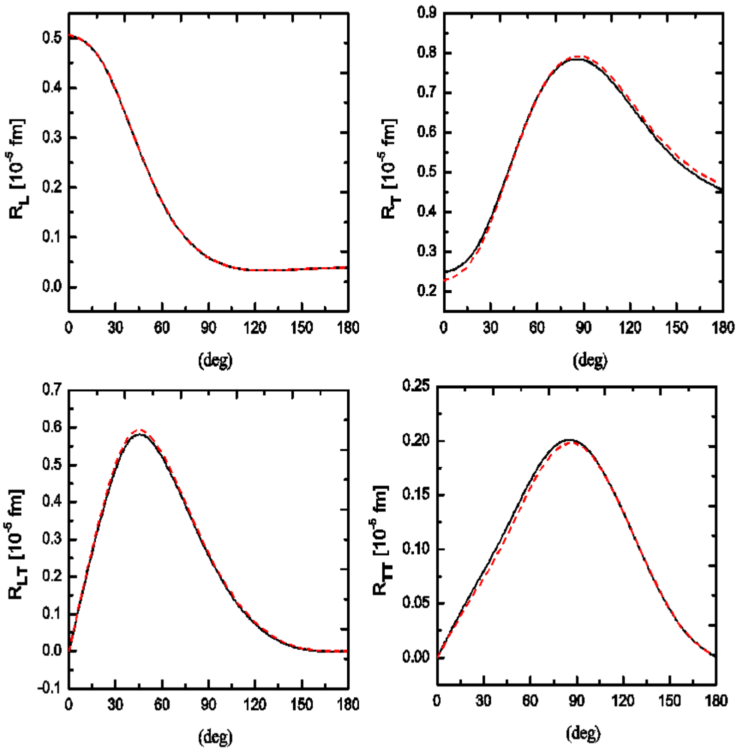

Figure 3. Angular dependence of the four semi-exclusive structure functions of at $k_0^{lab} = 300$ MeV and squared four-momentum transfer $Q^2 = 0.01 \text{ GeV}^2$, solid lines where the $\Omega(782)$ is included and dashed lines where $\Omega(782)$ is eliminated.

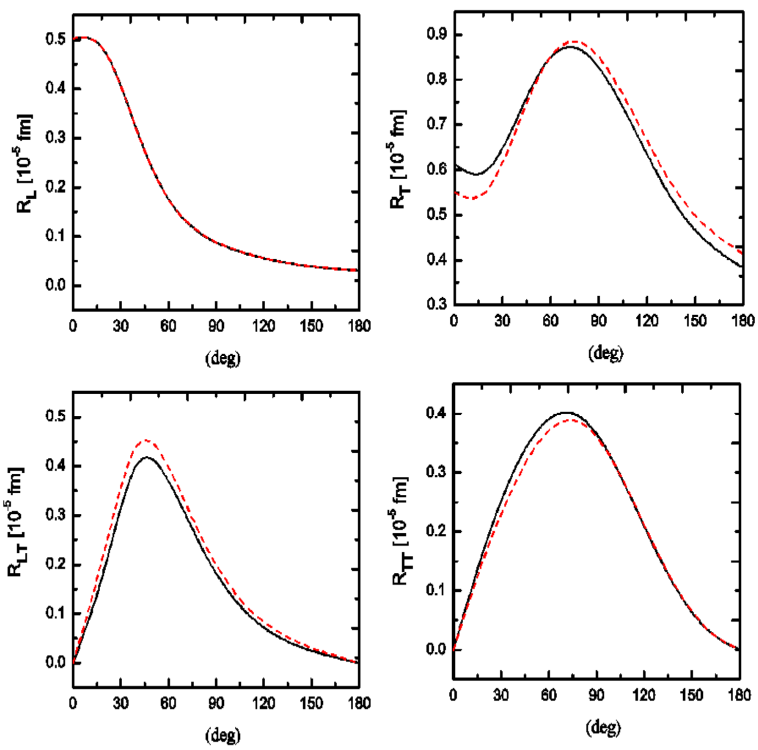

Figure 4. Notation as in Figure 3 at $k_0^{lab} = 350$ MeV.

In Figure 5 the virtual photon lab energy is increased to $k_0^{lab} = 400$ MeV at the same value of the four momentum transfer $Q^2 = 0.01 \text{ GeV}^2$, the contributions of $\Omega(782)$ are somewhat bigger than what was found in Figure 4 but still so very small.

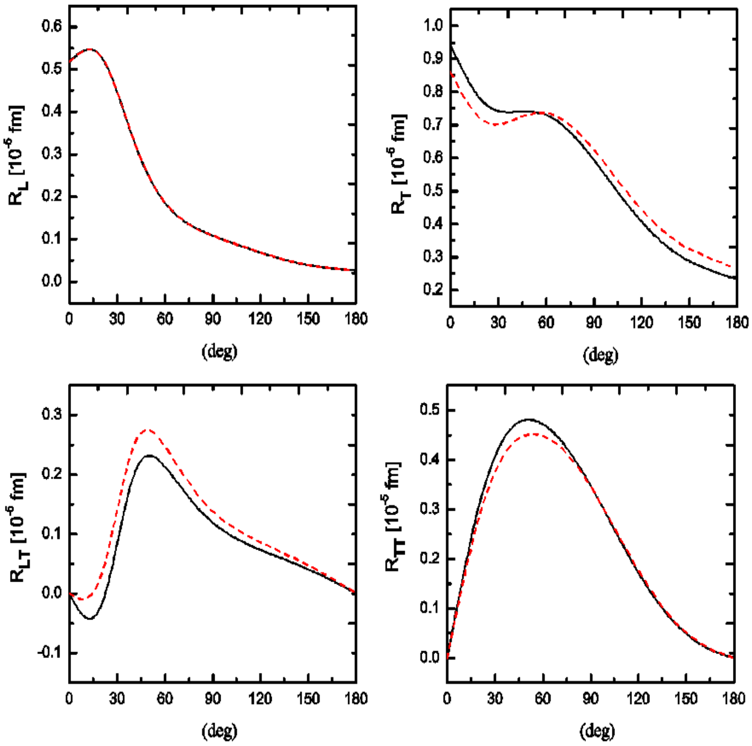

Figure 5. Notation as in Figure 3 at $k_0^{lab} = 400$ MeV.

In Figure 6, where the four momentum transfer $Q^2 = 0.05$ GeV^2 and the virtual photon lab energy $k_0^{lab} = 300$ MeV, the contribution of $\Omega(782)$ can be neglected for all structure functions.

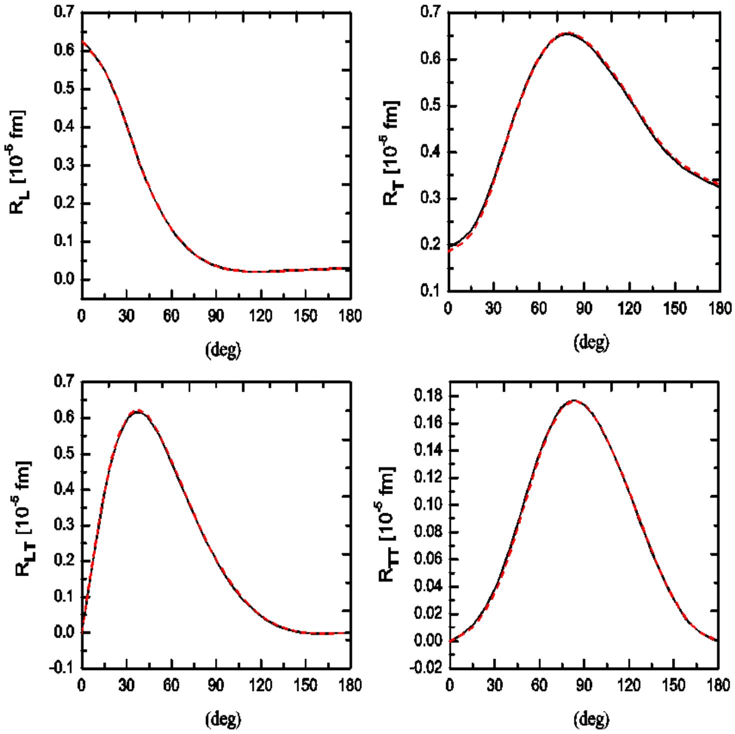

Figure 6. Notation as in Figure 3 at $Q^2 = 0.05$ GeV^2 .

Increasing virtual photon lab energy to $k_0^{lab} = 350$ MeV and keeping the four momentum transfer $Q2 = 0.05\ \mathrm{GeV}^2$ (Figure 7), the contributions of $\Omega(782)$ become more noticeable than in Figure 6 but still very small.

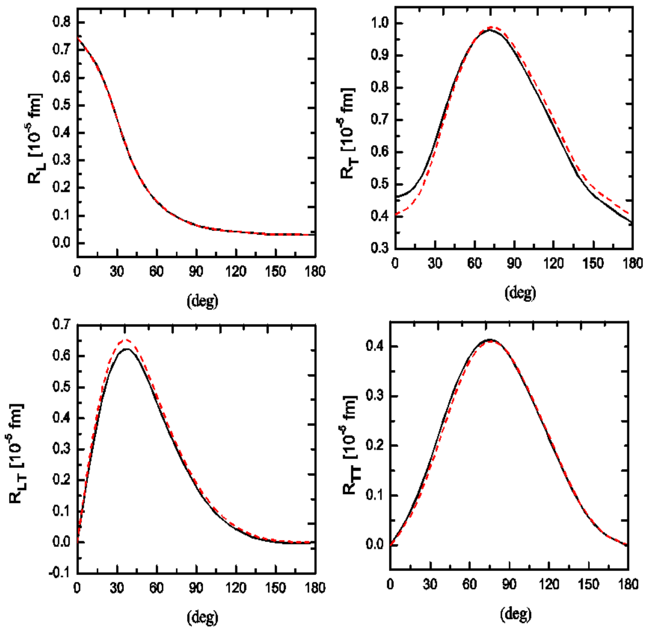

Figure 7. Notation as in Figure 6 at $k_0^{lab} = 350$ MeV.

In Figure 8 the virtual photon lab energy is increased to $k_0^{lab} = 400$ MeV at the same value of the four momentum transfer $Q^2 = 0.05\ \mathrm{GeV}^2$, the contributions of $\Omega(782)$ are somewhat bigger than in Figure 7 but also still so very small.

The same results are found for $Q^2 = 0.1\ \mathrm{GeV}^2$ at $k_0^{lab} = 300, 350$ and 400 MeV (see Figures 9-11). This mean, the effect of adding $\Omega(782)$ is very small and slightly increase with increasing the virtual photon lab energy. The four momentum transferee value plays no role in the effect of adding $\Omega(782)$ resonance.

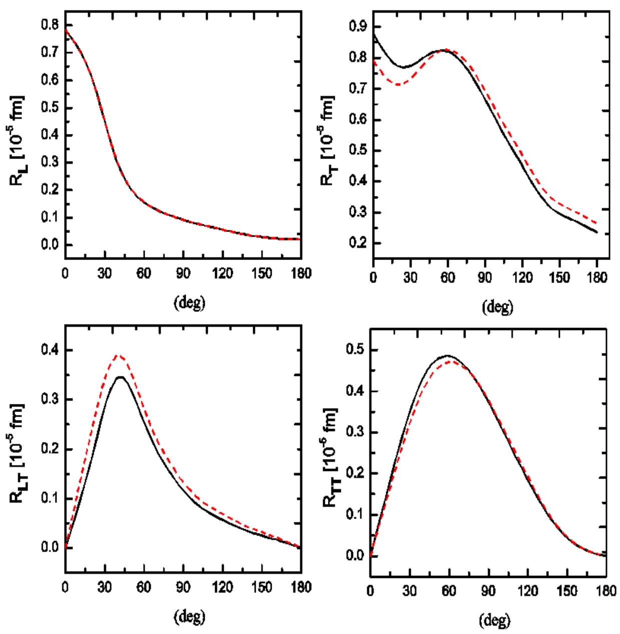

Figure 8. Notation as in Figure 6 at $k_0^{lab} = 400$ MeV.

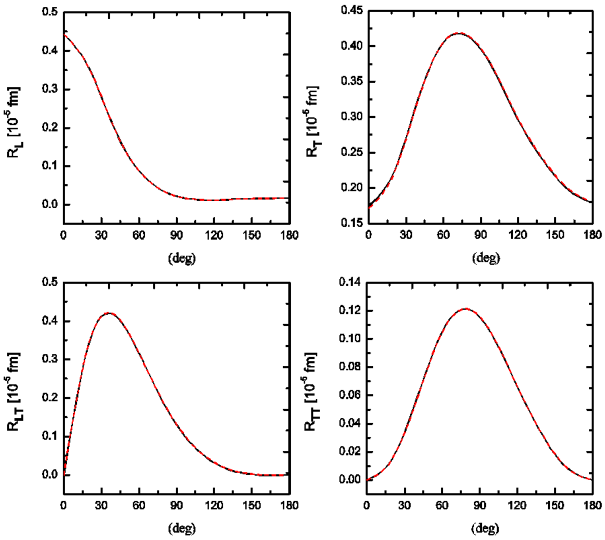

Figure 9. Notation as in Figure 3 at $Q^2 = 0.1\ \mathrm{GeV}^2$.

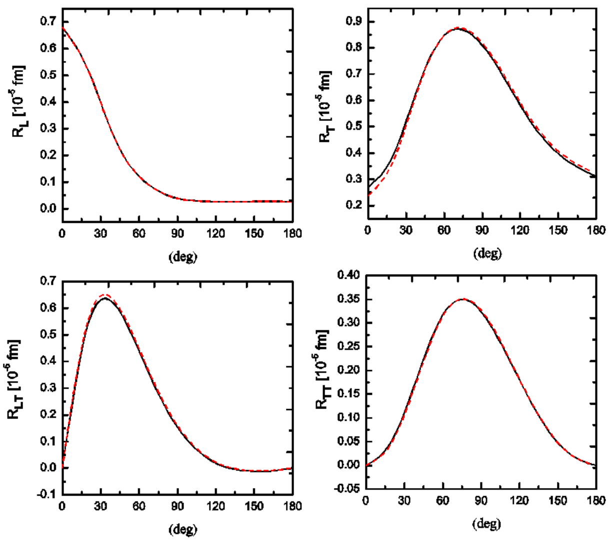

Figure 10. Notation as in Figure 9 at $k_0^{lab} = 350$ MeV.

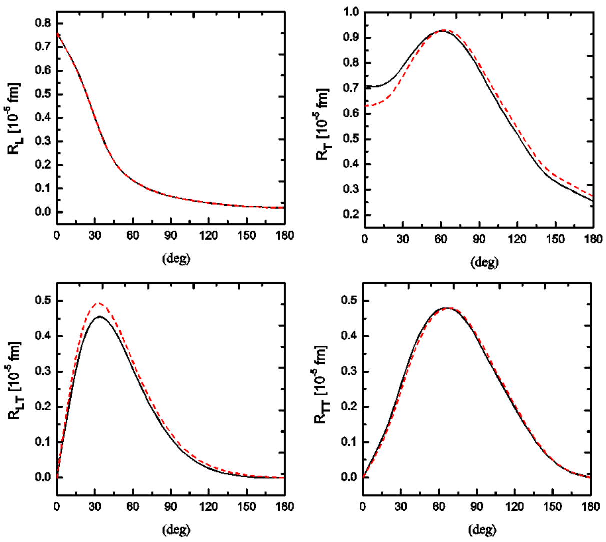

Figure 11. Notation as in Figure 9 at $k_0^{lab} = 400$ MeV.

Conclusions

A systematic study of the contribution of $\Omega(782)$ resonance to the structure functions of the positive pion electroproduction on the deuteron has been carried out. The study used the IA without final state interactions. As the structure functions depend on the squared four momentum transfer Q^2, the invariant energy or, equivalently, the virtual photon laboratory energy and the outgoing pion angle in the final hadronic c.m. system, three values for $\Omega(782)$ and k_0^{lab} have been selected for the presentation of the results. The results show a small effect of $\Omega(782)$ resonance on the positive pion electroproduction on the deuteron, this effect does not change when Q^2 increases and increases when k_0^{lab} increases.

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