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Neutrino oscillation induced by horizontal symmetry

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To obtain the interactions which cause neutrino flavor conversion, we introduce a horizontal symmetry into the standard model (SM) and propose the hypothesis that new interactions generated by the horizontal symmetry lead to neutrino flavor conversion and oscillation. To support our hypothesis, we evaluate the flavor conversion probability by new interactions by utilizing the definition of cross section, and the prediction is consistent to experimental data. From our hypothesis, neutrino oscillation is fluctuation of flavor distribution before arriving at equilibrium.

Keywords: neutrino-neutrino interaction, neutrino oscillation, beyond SM.

Introduction

The phase evolution can depict phenomenologically neutrino oscillation [1] very well, and also manifest neutrinos massive. But the interactions which cause neutrino flavor conversion remain puzzling [2,3], which we will attempt to investigate in this paper.

No interactions implied in the SM can induce neutrino flavor conversion, while a horizontal symmetry added into SM will provide these interactions [4] and also the masses of neutrinos [5]. Therefore we extend the SM and propose a hypothesis that new interactions from horizontal symmetry lead to neutrino flavor conversions. And neutrino oscillation is the macro phenomenon of the sum of all interactions before equilibrium, which will results in a mixing matrix of anarchy [6,7].

To support our hypothesis, we will calculate the flavor conversion probability by new interactions and compare it to the experimental data. In these calculation, we will assume neutrinos collide due to 'Brown movement' with their moments of all possible directions. We will uncover that it is their mass differences that lead to three different neutrino mixing angles.

This paper will be constructed as follows. We will introduce the model of $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$, which is an extension of SM by adding a horizontal symmetry $SU(2)_N$. We will evaluate the flavor conversion probability induced by new interactions and compare it with experimental data. We will demonstrate the resultant phenomenon of neutrino oscillation.

The model of $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$

For simplicity, we will take two-flavor frame. Thus, our model contains four leptons v_e, e, v_μ, μ and four quarks with neglecting the color degree of freedom. (For anomaly free, quark sector is assigned the same as SM and we will not discuss them. And we will not discuss the source of neutrino mass in this paper either.)

The total group is $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$. Considering suppression of $e \leftrightarrow \mu$ in experiments, the horizontal flavor symmetry $SU(2)_N$ only works between neutrinos. Thus the up sectors of SM doublets are under the horizontal symmetry $SU(2)_N$, while the lower sectors are not. We will assign the scale which breaking $SU(2)_N$ just a little higher than the scale of SM. After $SU(2)_N$ broken, the model should return to SM.

For 2×2 representation of field like ψ,the transformation under the group $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$ can be equivalent to transformation under group $SU(2)_L$, $U(1)_Y$ and $SU(2)_N$ in the meantime. (1) is the transformations of ψ under group $SU(2)_L$, $U(1)_Y$ and $SU(2)_N$ respectively:

$$\begin{gathered}\psi \to e^{-i\frac{1}{2}\alpha^a(x)\tau_w^a}\psi \\ \psi \to e^{-i\frac{1}{2}\beta(x)\hat{Y}}\psi \\ \psi \to \psi e^{i\frac{1}{2}\rho^b(x)\tau_F^b}\end{gathered} \tag{1}$$

In (1), τ_w, τ_F are generators associated with $SU(2)_L$ and $SU(2)_N$ respectively, $\hat{Y}$ is the weak hypercharge operator related to $U(1)_Y$. The covariant derivatives related with transformation of ψ under $SU(2)_L$, $U(1)_Y$ and $SU(2)_N$ respectively are

$$D_\mu = \partial_\mu - i\frac{1}{2}g_w \tau_w^a W_\mu^a \tag{2}$$

$$D_\mu = \partial_\mu - i\frac{1}{2}g_Y Y_\mu \hat{Y} \tag{3}$$

$$D_\mu \psi = \partial_\mu \psi + i\frac{1}{2}g_F \psi \tau_F^a F_\mu^a \tag{4}$$

In our model, two neutrino flavors are assigned into the doublet representation of $SU(2)_N$ as follows

$$\begin{pmatrix} v_e \\ v_\mu \end{pmatrix} \qquad (2,2,-1) \tag{5}$$

The numbers within parentheses stand for $SU(2)_L, SU(2)_N$ and $U(1)_Y$ quantum numbers ($2I+1, 2I_N+1, Y$) where I, I_N are isospins of the subgroups $SU(2)_L, SU(2)_N$ respectively, and Y is the $U(1)$ -hypercharge.

The $SU(2)_L \otimes U(1)_Y$ is $W-S$ model and the doublets are

$$\begin{pmatrix} v_e \\ e \end{pmatrix}_L, \qquad \begin{pmatrix} v_\mu \\ \mu \end{pmatrix}_L \tag{6}$$

and the lower sectors e_L, μ_L are assigned as $(2,1,-1)$.

We assign all leptons in a left-handed 2×2 matrix

$$\begin{pmatrix} v_e & v_\mu \\ e & \mu \end{pmatrix}_L \tag{7}$$

and two right handed singlets

$$R_e = e_R, \qquad R_\mu = \mu_R \qquad (1,1,-2) \tag{8}$$

The electric charge formula is given by

$$Q = I_{3L} + Y/2 \tag{9}$$

Thus, there are seven vector bosons W_α^i, F_α^i $(i = 1 \sim 3)$ and Y_α, associated with the subgroups $SU(2)_L, SU(2)_N, U(1)_Y$ respectively.

The fermion dynamical Lagrangian is

$$\begin{aligned} \mathcal{L}_f = i\bar{R}_e\gamma^\alpha(\partial_\alpha + ig_Y Y_\alpha)R_e + i\bar{R}_\mu\gamma^\alpha(\partial_\alpha + ig_Y Y_\alpha)R_\mu \\ + iTr[\bar{L}\gamma^\alpha(\partial_\alpha + i\frac{1}{2}g_Y Y_\alpha - i\frac{1}{2}g_w \tau_w^i W_\alpha^i)L] \\ + iTr[\bar{L}\gamma^\alpha i\frac{1}{2}g_F\frac{1}{2}(1+\tau_3)L\tau_F^i F_\alpha^i)] \end{aligned} \tag{10}$$

where

$$\tau_w^i W_\mu^i = \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^- \\ \sqrt{2}W_\mu^+ & -W_\mu^3 \end{pmatrix} \tag{11}$$

$$\tau_F^i F_\mu^i = \begin{pmatrix} F_\mu^3 & \sqrt{2}F_\mu^+ \\ \sqrt{2}F_\mu^- & -F_\mu^3 \end{pmatrix} \tag{12}$$

The difference of (10) from [4] is that τ_F^i only act on neutrino fields and vector bosons F_α^i only intermediate neutrino-neutrino interactions, and the term $\frac{1}{2}(1+\tau_3)$ selects the up sectors of L for horizontal symmetry $SU(2)_N$. The proof of the fermion dynamical Lagrangian (10) invariant under the group $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$ is shown in Appendix A.

Though our horizontal symmetry exists just between neutrinos, the masses of vector bosons are given by Higgs scalars vacuum expectation value (VEV) and the mass matrix (13) for W^3, F^3, Y and the diagonal vectors (14) in [4] are also suitable for our model

$$\begin{pmatrix} g_w^2 B & g_w g_F C & g_w g_Y B \\ g_w g_F C & g_F^2 D & g_Y g_F C \\ g_w g_Y B & g_Y g_F C & g_Y^2 B \end{pmatrix} \tag{13}$$

The diagonal neutral vectors are

$$\begin{aligned} A^\mu &= \frac{1}{N_A}(g_Y W_3^\mu - g_w Y^\mu) \\ Z^\mu &= \frac{1}{N_Z}(g_Y Y^\mu + g_w W_3^\mu + g_F Y_1 F_3^\mu) \\ G^\mu &= \frac{1}{N_G}(g_Y Y^\mu + g_w W_3^\mu + g_F Y_2 F_3^\mu) \end{aligned} \tag{14}$$

For conveniently comparing with the Weinberg angle in SM, we assume

$$\begin{aligned} g_F &= g cos\theta \\ g_Y &= g sin\theta sin\phi \\ g_w &= g sin\theta cos\phi \end{aligned} \tag{15}$$

The interaction Lagrangian can then be written as

$$\begin{aligned}
\mathcal{L}_{int} = \frac{\sqrt{2}}{2} g_w [W_\alpha^- (\bar{e}_L \gamma^\alpha v_e + \bar{\mu}_L \gamma^\alpha v_\mu) + W_\alpha^+ (\bar{v}_e \gamma^\alpha e_L + \bar{v}_\mu \gamma^\alpha \mu_L)] \\
-\frac{\sqrt{2}}{2} g_F [F_\alpha^- \bar{v}_e \gamma^\alpha v_\mu + F_\alpha^+ \bar{v}_\mu \gamma^\alpha v_e] \\
+ g sin\theta sin\phi cos\phi A_\alpha (\bar{e}\gamma^\alpha e + \bar{\mu}\gamma^\alpha \mu) \\
+\frac{y_2}{y_2 - y_1}(cos^2\phi - sin^2\phi)(\bar{e}_L \gamma^\alpha e_L + \bar{\mu}_L \gamma^\alpha \mu_L) \cdot N_Z Z_\alpha \\
+\frac{1}{2}(\frac{1-y_2}{y_2-y_1}\bar{v}_e \gamma^\alpha v_e - \frac{1+y_2}{y_2-y_1}\bar{v}_\mu \gamma^\alpha v_\mu) \cdot N_Z Z_\alpha \\
+\frac{y_1}{y_2-y_1} sin^2\phi (\bar{R}_e \gamma^\alpha R_e + \bar{R}_\mu \gamma^\alpha R_\mu) \cdot N_G G_\alpha \\
+\frac{1}{2}\frac{y_1}{y_2-y_1}(sin^2\phi - cos^2\phi)(\bar{e}_L \gamma^\alpha e_L + \bar{\mu}_L \gamma^\alpha \mu_L) \cdot N_G G_\alpha \\
+\frac{1}{2}(\frac{y_1}{y_2-y_1} sin^2\phi - \frac{1}{y_2-y_1})\bar{v}_e \gamma^\alpha v_e \cdot N_G G_\alpha \\
+\frac{1}{2}(\frac{y_1}{y_2-y_1} sin^2\phi + \frac{1}{y_2-y_1})\bar{v}_\mu \gamma^\alpha v_\mu \cdot N_G G_\alpha
\end{aligned} \tag{16}$$

The coupling constants in (16) compared with the ones in SM will yield

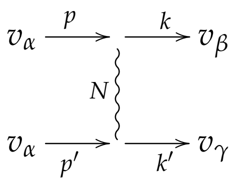
$$\begin{aligned}
e = g sin\theta sin\phi cos\phi \\
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_w^2} sin^2\theta cos^2\phi
\end{aligned} \tag{17}$$

Comparing the third formula in (15) and the second formula in (17), we can deduce that g_w in our model is just the one in SM. Combined with the first formula in (17), we can deduce that ϕ is just the Weinberg angle θ_W in SM. The maximal mixture between neutrinos implies the coupling constant g_F for interactions between different neutrinos is similar to g_Z in SM for the same flavors. Then we assume the magnitude of g_F is the same as g_Z in SM.

The flavor conversion probability per unit time

The cross section σ and the scattering probability per unit time at the start of neutrino flight

We assume neutrinos collide in beam due to their 'Brownian movement' with their moments of all possible directions. According to our model, neutrino flavor conversions are caused by interactions intermediated by F_μ. At the start of neutrino's flight, all neutrino-neutrino interactions can be shown in one Feynman diagram as (18)



$$\alpha, \beta, \gamma = e, \mu, \tau \tag{18}$$

v_α is the initial neutrino and N is vector boson which intermediates neutrino-neutrino interactions such as $Z, F^{\pm}$. We assume these vector bosons have masses similar to Z^0 in SM.

For small mass of neutrino, our following calculation will ignore neutrino mass. Thus, the scattering amplitude of (18) is

$$iM = \frac{-ig_N^2}{M_N^2}[\bar{u}_{v_\beta}(k)\gamma^\mu(1-\gamma_5)u_{v_\alpha})(p)][\bar{u}_{v_\gamma}(k')\gamma_\mu(1-\gamma_5)u_{v_\alpha}(p')] \tag{19}$$

$$m_{v_e} \sim m_{v_\mu} \sim m_{v_\tau} \sim 0 \tag{20}$$

As is mentioned, the magnitude of $g_F \sim g_Z(inSM)$ and then

$$\sigma_{CM} = \frac{\overline{|M|^2}}{16\pi E_{cm}^2} = \frac{4^3 \times g_z^4}{16\pi E_{cm}^2 M_N^4}(k \cdot k')(p \cdot p') = \frac{4 \times g_z^4}{\pi E_{cm}^2 M_N^4}(k \cdot k')(p \cdot p') \tag{21}$$

Not that (21) is obtained with the convention that the impact parameter $b \sim 0$. The neutrino conversion probability generally is obtained on axis, so (21) makes sense.

In center of mass frame , we have

$$k \cdot k' = \tfrac{1}{2}(2k \cdot k') = \tfrac{1}{2}[(k+k')^2 - (m_\beta^2 + m_\gamma^2)] = \tfrac{1}{2}[E_{cm}^2 - (m_\beta^2 + m_\gamma^2)]$$
$$p \cdot p' = \tfrac{1}{2}(2p \cdot p') = \tfrac{1}{2}[(p+p')^2 - 2m_\alpha^2] = \tfrac{1}{2}[E_{cm}^2 - 2m_\alpha^2]$$

And we get scattering cross section in center of mass frame as below

$$\sigma_{CM} = \frac{4 \times g_z^4}{\pi E_{CM}^2 M_N^4}(k \cdot k')(p \cdot p') = \frac{g_z^4 \times [E_{cm}^2 - (m_\beta^2 + m_\gamma^2)] \times [E_{cm}^2 - 2m_\alpha^2]}{\pi E_{CM}^2 M_N^4} \approx \frac{g_z^4 \times E_{CM}^2}{\pi M_N^4} \tag{22}$$

Let

$$s = (p+p')^2 = (k+k')^2 = E_{cm}^2 \tag{23}$$

We can obtain the expression about s of section σ

$$\sigma \approx \frac{g_z^4}{\pi M_z^4} \times s \qquad (M_N \approx M_Z) \tag{24}$$

Taking the rest frame of the initial neutrino $v_\alpha(p)$ before interaction, we have

$$s = 2p \cdot p' + 2m_{v_\alpha}^2 \approx 2p \cdot p' \approx 2m_{v_\alpha} E_{v_\alpha} \tag{25}$$

Then we have scattering cross section as below

$$\sigma \approx \frac{g_z^4}{\pi M_z^4} \times s \approx \frac{g_z^4 \times 2m_{v_\alpha} E_{v_\alpha}}{\pi M_z^4} \tag{26}$$

After some calculation of natural units conversion (which we will show in Appendix B), we get

$$\sigma \approx \frac{10^{-19}}{16\pi}(GeV)^{-2} = \frac{1}{4\pi} \times 10^{-47} cm^2 \tag{27}$$

Here we assume $m_{v_\alpha} = m_{v_e} \sim 1eV$ and the energy of neutrino $E = 1GeV$. Generally, the masses of the former two neutrino families v_e, v_μ are considered far lighter than the third v_τ. Thus when v_τ participates in interactions, its mass in (22) can not be ignored and (27) is not suitable for these interactions and therefore the cross section of these interactions should obtained from (22) directly. In this paper, we only want to know whether our predicting conversion probability is consistent to experiments. For simplicity and without loss of generality, we only need to evaluate the order of predicting $P(v_e \to v_\mu)$.

Following, we will show how to evaluate conversion probability by cross section σ. Consider the definition of σ:

$$\sigma = \frac{N}{n_B \cdot N_A} = \frac{1}{n_B \cdot N_A} \int d^2b \; n_B P(b). \tag{28}$$

In equation(28), n_B denotes current density of incident particle B; N_A denotes particle number of target particle A; N denotes all scattering events in unit time.

From (28), if beam section is unit area, the value of current density n_B equals incident particle number in unit time. In neutrino beam , target particle is also particles in beam of the same section and the number of target particle $N_A = n_B$. Thus , according to (28), the value of σ equals the average scattering probability per neutrino in unit time. That means the value of σ equals the value of average conversion probability per unit time of one initial neutrino. Because the cross section formula (21) is established with the convention $b \sim 0$, we will choose the unit area $1cm^2$. Then the magnitude of neutrino-neutrino scattering probability per unit time is

$$P(v_\alpha + v_\alpha \to v_\beta + v_\gamma) = \frac{1}{4\pi} \times 10^{-47} \times 2 \tag{29}$$

The factor 2 is because 2 initial neutrinos.

Thus our predicting neutrino flavor conversion probability per unit time is

$$P(v_\alpha \to v_\beta) \sim O(10^{-47}) \tag{30}$$

The actual conversion probability per unit time at the start of flight

We can obtain the conversion probability per time (32) by taking a derivative of the empirical formula (31) with respect to t:

$$P(v_\alpha \to v_\beta) = sin^2(2\theta_{ij})sin^2(\frac{t}{L_0}) \qquad L_0 = \frac{4E}{\Delta m_{ij}^2} \tag{31}$$

$$\frac{dP(v_\alpha \to v_\beta)}{dt} = 2sin^2(2\theta_{ij})sin(\frac{t}{L_0})cos(\frac{t}{L_0})\frac{1}{L_0} \approx 2sin^2(2\theta_{ij})\frac{t}{L_0^2} \tag{32}$$

The last term of (32) is because $t \ll L_0$ at the start of flight.

In the first second of flight,

$$\frac{dP(v_\alpha \to v_\beta)}{dt} \approx 2sin^2(2\theta_{ij})\frac{t}{L_0^2} = 2sin^2(2\theta_{ij})\frac{1}{L_0^2} = 2sin^2(2\theta_{ij})(\frac{\Delta m_{ij}^2}{4E})^2 \tag{33}$$

To estimate the magnitude of (33), we take a special example as the conversion of $v_e \to v_\mu$. According to experimental data, we take values of (34) into (33):

$$\begin{aligned} &E \approx 1GeV \\ &\Delta m_{12}^2 \approx 8 \times 10^{-5}(eV)^2 = 8 \times 10^{-23}(GeV)^2 \\ &sin^2(2\theta_{12}) = 0.86 \end{aligned} \tag{34}$$

So, the conversion probability per unit time at the start of neutrino flight is

$$P(v_e \to v_\mu) \approx 2sin^2(2\theta_{12})(\frac{8 \times 10^{-23}}{4})^2 \approx 6 \times 10^{-46} \tag{35}$$

Apparently, our prediction (30) is very close to the actual value(35).

Neutrino oscillation

According to our model, neutrino-neutrino interaction is

$$\mathcal{L}_{int} = -ig'_Z \sum_{\alpha} \bar{v_\alpha}\gamma^\mu(1-\gamma_5)v_\alpha Z_\mu - ig_F \sum_{\alpha\neq\beta} \bar{v_\beta}\gamma^\mu(1-\gamma_5)v_\alpha F^{\pm}_\mu \tag{36}$$

As is mentioned before, we assume

$$\begin{matrix} g'_Z \sim g_Z \sim g_F \\ M_Z \sim M_F \end{matrix}\;, \tag{37}$$

then the whole approximate effective Hamiltonian can be written in one term as follows

$$H_{int} = \frac{G'_F}{\sqrt{2}} \int dx^3 (\sum_i \bar{v}'_{iL}\gamma_\mu v_{iL})(\sum_j \bar{v}'_{jL}\gamma^\mu v_{jL}) \tag{38}$$

Here v' is not a pure state and is some mixture of several states. While v_{iL}, v_{jL} are initial states. Thus the neutrino mixture is just the macro result of the sum of all interactions. And we also can conclude that the mixture of neutrino flavors will not be constant until all neutrino-neutrino interactions arrive at balance.

We can see that there is a equilibrium state for these interactions. There must have some oscillations before neutrino probability distribution arrive at equilibrium distribution.

Conclusions

For interactions responsible for neutrino flavor conversions, we constructed the extension of SM including a horizontal flavor symmetry i.e. $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$. The horizontal symmetry $SU(2)_N$ introduces interactions between different neutrino flavors and leads to neutrino flavor conversions.

To testify our idea, we have evaluated the flavor conversion probability induced by new interactions by utilizing the definition of cross section. In this calculation, we assume neutrinos collide due to 'Brown movement' and the direction of individual neutrino instantaneous moment in beam is not definite and generally not consistent to the direction of neutrino flux. Fortunately, the prediction is consistent with experimental data. And we also demonstrate neutrino oscillation as phenomenon before all neutrino-neutrino interactions arriving at balance.

Appendix

A. The invariance of the fermion dynamical Lagrangian under the group $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$

The invariance of L transforming under the group $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$ can be equivalent to invariance under group $SU(2)_L$, $U(1)_Y$ and $SU(2)_N$ in the meantime:

$$\begin{aligned}
&L \to e^{-i\frac{1}{2}\alpha^{a}(x)\tau_{w}^{a}}L \\
&L \to e^{-i\frac{1}{2}\beta(x)\hat{Y}}L \quad (A.1) \\
&L \to Le^{i\frac{1}{2}\rho^{b}(x)\tau_{F}^{b}}
\end{aligned}$$

α, β, ρ is infinitesimal, τ_w, τ_F are generators associated with $SU(2)_L$ and $SU(2)_N$ respectively, $\hat{Y}$ is the weak hypercharge operator related to $U(1)_Y$. We introduce gauge fields $W_\alpha^i, Y_\mu, F_\alpha^i$ to relate groups $SU(2)_L$, $U(1)_Y$ and $SU(2)_N$ respectively.

The covariant derivatives related with transformation under $SU(2)_L$, $U(1)_Y$ and $SU(2)_N$ respectively are

$$\begin{aligned}
&D_\mu = \partial_\mu - i\tfrac{1}{2}g_w\tau_w^a W_\mu^a \\
&D_\mu = \partial_\mu - i\tfrac{1}{2}g_Y Y_\mu \hat{Y} \qquad (A.2) \\
&D_\mu L = \partial_\mu L + i\tfrac{1}{2}g_F\tau_F^a F_\mu^a
\end{aligned}$$

Define the gauge transformation of gauge fields $W_\alpha^i, Y_\mu, F_\alpha^i$ as follows

$$\begin{aligned}
&ig_w\tau^a W_\mu^a = ig_w\tau^a W_\mu^a - i\partial_\mu\alpha^b\tau^b - \tfrac{1}{2}g_w[\tau^a W_\mu^a, \alpha^b\tau^b] \\
&Y_\mu = Y_\mu - \tfrac{1}{g_Y}\partial_\mu\beta \qquad (A.3) \\
&ig_F\tau^a F_\mu^a = ig_F\tau^a F_\mu^a - i\partial_\mu\rho^b\tau^b - \tfrac{1}{2}g_F[\tau^a F_\mu^a, \rho^b\tau^b]
\end{aligned}$$

The fermion dynamical Lagrangian invariant under the group $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$ is

$$\begin{aligned}
&\mathcal{L}_f = i\bar{R}_e\gamma^\alpha(\partial_\alpha + ig_Y Y_\alpha)R_e + i\bar{R}_\mu\gamma^\alpha(\partial_\alpha + ig_Y Y_\alpha)R_\mu \\
&+iTr[\bar{L}\gamma^\alpha(\partial_\alpha + i\tfrac{1}{2}g_Y Y_\alpha - i\tfrac{1}{2}g_w\tau_w^i W_\alpha^i)L] \qquad (A.4) \\
&+iTr[\bar{L}\gamma^\alpha i\tfrac{1}{2}g_F\tfrac{1}{2}(1+\tau_3)L\tau_F^i F_\alpha^i)]
\end{aligned}$$

In (A.4), for left-handed lepton, the value of the hypercharge Y is -1 and for right-handed lepton R, the value of the hypercharge Y is -2.

With the transformation of these gauge fields (42), the fermion dynamical Lagrangian (A.4) will be invariant under the group of $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$. We will prove it term by term.

Proof:

1. The first two terms in (A.4)

$$R_e \to e^{-i\frac{1}{2}\beta(x)\hat{Y}}R_e \to (1 - i\frac{1}{2}\beta(x)\hat{Y})R_e = (1 + i\beta(x))R_e$$

The covariant derivative is

$$D_\mu = \partial_\mu + ig_Y Y_\mu$$

and the transformation of gauge field

$$Y_\mu \to Y_\mu - \frac{1}{g_Y}\partial_\mu \beta$$

We only need to prove

$$D_\mu(1+i\beta(x))R_e \to (1+i\beta(x))D_\mu R_e$$

i.e.

$$(\partial_\mu + ig_Y Y_\mu)[(1+i\beta(x))R_e] \to (1+i\beta(x))(\partial_\mu + ig_Y Y_\mu)R_e \qquad (A.5)$$

Left of (A.5) =

$$\begin{aligned}
&i\partial_\mu\beta R_e + (1+i\beta)\partial_\mu R_e + ig_Y Y_\mu R_e - g_Y Y_\mu \beta R_e\\
&\to i\partial_\mu\beta R_e + (1+i\beta)\partial_\mu R_e + ig_Y(Y_\mu - \tfrac{1}{g_Y}\partial_\mu\beta)R_e - g_Y(Y_\mu - \tfrac{1}{g_Y}\partial_\mu\beta)\beta R_e\\
&\to \partial_\mu R_e + i\partial_\mu\beta R_e + i\beta\partial_\mu R_e + ig_Y Y_\mu R_e - i\partial_\mu\beta R_e - g_Y Y_\mu\beta R_e + \partial_\mu\beta\beta R_e\\
&\to \partial_\mu R_e + i\beta\partial_\mu R_e + ig_Y Y_\mu R_e - g_Y Y_\mu\beta R_e + \partial_\mu\beta\beta R_e
\end{aligned}$$

Right =

$$\begin{aligned}
&(1+i\beta(x))(\partial_\mu + ig_Y Y_\mu)R_e\\
&\to \partial_\mu R_e + i\beta\partial_\mu R_e + ig_Y Y_\mu R_e - g_Y Y_\mu\beta R_e
\end{aligned}$$

Omitting the higher order of $O(g\beta)$, we can obtain

$$(\partial_\mu + ig_Y Y_\mu)[(1+i\beta(x))R_e] \to (1+i\beta(x))(\partial_\mu + ig_Y Y_\mu)R_e$$

2. The third and fourth term

We will prove the invariance of these two terms by prove invariance under group $SU(2)_L$, $U(1)_Y$ and $SU(2)_N$ respectively.

The transformations of the gauge fields are

$$\begin{aligned}
&\tau^a W^a_\mu \to \tau^a W^a_\mu - \tfrac{1}{g_w}\partial_\mu\alpha^b\tau^b + i\tfrac{1}{2}[\tau^a W^a_\mu, \alpha^b\tau^b]\\
&Y_\mu \to Y_\mu - \tfrac{1}{g_Y}\partial_\mu\beta\\
&\tau^a F^a_\mu \to \tau^a F^a_\mu - \tfrac{1}{g_F}\partial_\mu\rho^b\tau^b + i\tfrac{1}{2}[\tau^a F^a_\mu, \rho^b\tau^b]
\end{aligned}$$

(1) (A.4) is invariant under $U(1)_Y$

$$L \to [1 + i\tfrac{1}{2}\beta(x)]L$$
$$D_\mu = \partial_\mu + i\tfrac{1}{2}g_Y Y_\mu$$

Which we need to prove is

$$L \to [1 + i\tfrac{1}{2}\beta(x)]L$$
$$D_\mu[1 + i\tfrac{1}{2}\beta(x)]L \to [1 + i\tfrac{1}{2}\beta(x)]D_\mu L,$$

that is

$$(\partial_\mu + i\frac{1}{2}g_Y Y_\mu)[1 + i\frac{1}{2}\beta(x)]L \to [1 + i\frac{1}{2}\beta(x)](\partial_\mu + i\frac{1}{2}g_Y Y_\mu)L \qquad (A.6)$$

with

$$Y_\mu \to Y_\mu - \frac{1}{g_Y}\partial_\mu\beta$$

Left of (A.6) =

$$\begin{aligned}
&(\partial_\mu + i\tfrac{1}{2}g_Y Y_\mu)[1 + i\tfrac{1}{2}\beta(x)]L \\
&= i\tfrac{1}{2}\partial_\mu\beta(x)L + [1 + i\tfrac{1}{2}\beta(x)]\partial_\mu L + i\tfrac{1}{2}g_Y Y_\mu L - \tfrac{1}{4}g_Y Y_\mu \beta L \\
&\to i\tfrac{1}{2}\partial_\mu\beta(x)L + [1 + i\tfrac{1}{2}\beta(x)]\partial_\mu L + i\tfrac{1}{2}g_Y(Y_\mu - \tfrac{1}{g_Y}\partial_\mu\beta)L - \tfrac{1}{4}g_Y(Y_\mu - \tfrac{1}{g_Y}\partial_\mu\beta)\beta L \\
&\approx [1 + i\tfrac{1}{2}\beta(x)]\partial_\mu L + i\tfrac{1}{2}g_Y Y_\mu L
\end{aligned}$$

The right of (A.6) is

$$\begin{aligned}
&[1 + i\tfrac{1}{2}\beta(x)](\partial_\mu + i\tfrac{1}{2}g_Y Y_\mu)L, \\
&= [1 + i\tfrac{1}{2}\beta(x)]\partial_\mu L + i\tfrac{1}{2}g_Y Y_\mu L - \tfrac{1}{4}\beta g_Y Y_\mu L \\
&\approx [1 + i\tfrac{1}{2}\beta(x)]\partial_\mu L + i\tfrac{1}{2}g_Y Y_\mu L
\end{aligned}$$

Thus (A.6) can be established.

(2) (A.4) is invariant under $SU(2)_L$

$$L \to [1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]L$$
$$D_\mu = \partial_\mu - i\tfrac{1}{2}g_w\tau^a W_\mu^a$$

Which we need to prove is

$$D_\mu[1 - i\frac{1}{2}\alpha^a(x)\tau^a]L = [1 - i\frac{1}{2}\alpha^a(x)\tau^a]D_\mu L,$$

that is

$$(\partial_\mu - i\frac{1}{2}g_w\tau^a W^a_\mu)[1 - i\frac{1}{2}\alpha^a(x)\tau^a]L = [1 - i\frac{1}{2}\alpha^a(x)\tau^a](\partial_\mu - i\frac{1}{2}g_w\tau^a W^a_\mu)L \qquad (A.7),$$

with

$$\tau^a W^a_\mu \to \tau^a W^a_\mu - \frac{1}{g_w}\partial_\mu\alpha^b\tau^b + i\frac{1}{2}[\tau^a W^a_\mu, \alpha^b\tau^b]$$

The left of (A.7) is

$$\begin{aligned}
&(\partial_\mu - i\tfrac{1}{2}g_w\tau^a W^a_\mu)[1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]L \\
&= -i\tfrac{1}{2}\partial_\mu\alpha^a(x)\tau^a L + [1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]\partial_\mu L - i\tfrac{1}{2}g_w\tau^a W^a_\mu L - \tfrac{1}{4}g_w\tau^a W^a_\mu\alpha^a(x)\tau^a L \\
&\to -i\tfrac{1}{2}\partial_\mu\alpha^a(x)\tau^a L + [1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]\partial_\mu L - i\tfrac{1}{2}g_w\{\tau^a W^a_\mu - \tfrac{1}{g_w}\partial_\mu\alpha^b\tau^b + i\tfrac{1}{2}[\tau^a W^a_\mu, \alpha^b\tau^b]\}L \\
&-\tfrac{1}{4}g_w\{\tau^a W^a_\mu - \tfrac{1}{g_w}\partial_\mu\alpha^b\tau^b + i\tfrac{1}{2}[\tau^a W^a_\mu, \alpha^b\tau^b]\}\alpha^c(x)\tau^c L \\
&= -i\tfrac{1}{2}\partial_\mu\alpha^a(x)\tau^a L + [1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]\partial_\mu L - i\tfrac{1}{2}g_w\tau^a W^a_\mu L + i\tfrac{1}{2}\partial_\mu\alpha^b\tau^b L \\
&+\tfrac{1}{4}g_w[\tau^a W^a_\mu, \alpha^b\tau^b]L - \tfrac{1}{4}g_w\tau^a W^a_\mu\alpha^b\tau^b L \\
&= [1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]\partial_\mu L - i\tfrac{1}{2}g_w\tau^a W^a_\mu L + \tfrac{1}{4}g_w[\tau^a W^a_\mu, \alpha^b\tau^b]L - \tfrac{1}{4}g_w\tau^a W^a_\mu\alpha^b\tau^b L \\
&= [1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]\partial_\mu L - i\tfrac{1}{2}g_w\tau^a W^a_\mu L - \tfrac{1}{4}g_w\alpha^b\tau^b\tau^a W^a_\mu L
\end{aligned}$$

Here, we omitting the higher orders.

The right of (A.7)=

$$\begin{aligned}
&[1 - i\tfrac{1}{2}\alpha^a(x)\tau^a](\partial_\mu - i\tfrac{1}{2}g_w\tau^a W^a_\mu)L \\
&= [1 - i\tfrac{1}{2}\alpha^a(x)\tau^a]\partial_\mu L - i\tfrac{1}{2}g_w\tau^a W^a_\mu L - \tfrac{1}{4}g_w\alpha^b\tau^b\tau^a W^a_\mu L
\end{aligned}$$

Thus (A.7) can be established.

(3) (A.4) is invariant under the horizontal group $SU(2)_N$

$$L \to L' = L(1 + i\frac{1}{2}\rho^b(x)\tau^b)$$
$$D_\mu L = \partial_\mu L + i\frac{1}{2}g_F L\tau^a F^a_\mu$$

Which we need to prove is

$$D_\mu[L(1+i\frac{1}{2}\rho^b(x)\tau^b)] \rightarrow (D_\mu L)(1+i\frac{1}{2}\rho^b(x)\tau^b),$$

that is

$$\begin{array}{l}\partial_\mu[L(1+i\frac{1}{2}\rho^b(x)\tau^b)] + i\frac{1}{2}g_F L(1+i\frac{1}{2}\rho^b(x)\tau^b)\tau^a F^a_\mu \\ \rightarrow (\partial_\mu L + i\frac{1}{2}g_F L\tau^a F^a_\mu)(1+i\frac{1}{2}\rho^b(x)\tau^b)\end{array} \qquad (A.8)$$

with

$$\tau^a F^a_\mu \rightarrow \tau^a F^a_\mu - \frac{1}{g_F}\partial_\mu\rho^b\tau^b + i\frac{1}{2}[\tau^a F^a_\mu, \rho^b\tau^b]$$

The left of (A.8) is

$$\begin{array}{l}\partial_\mu[L(1+i\frac{1}{2}\rho^b(x)\tau^b)] + i\frac{1}{2}g_F L(1+i\frac{1}{2}\rho^b(x)\tau^b)\tau^a F^a_\mu \\ = \partial_\mu L(1+i\frac{1}{2}\rho^b(x)\tau^b) + Li\frac{1}{2}\partial_\mu\rho^b(x)\tau^b + i\frac{1}{2}g_F L\tau^a F^a_\mu - \frac{1}{4}g_F L\rho^b(x)\tau^b\tau^a F^a_\mu \\ \rightarrow \partial_\mu L(1+i\frac{1}{2}\rho^b(x)\tau^b) + Li\frac{1}{2}\partial_\mu\rho^b(x)\tau^b + i\frac{1}{2}g_F L\{\tau^a F^a_\mu - \frac{1}{g_F}\partial_\mu\rho^b\tau^b + i\frac{1}{2}[\tau^a F^a_\mu, \rho^b\tau^b]\} \\ -\frac{1}{4}g_F L\rho^c(x)\tau^c\{\tau^a F^a_\mu - \frac{1}{g_F}\partial_\mu\rho^b\tau^b + i\frac{1}{2}[\tau^a F^a_\mu, \rho^b\tau^b]\} \\ \approx \partial_\mu L(1+i\frac{1}{2}\rho^b(x)\tau^b) + Li\frac{1}{2}\partial_\mu\rho^b(x)\tau^b + i\frac{1}{2}g_F L\tau^a F^a_\mu - i\frac{1}{2}L\partial_\mu\rho^b\tau^b \\ -\frac{1}{4}g_F L[\tau^a F^a_\mu, \rho^b\tau^b] - \frac{1}{4}g_F L\rho^b\tau^b\tau^a F^a_\mu \\ = \partial_\mu L(1+i\frac{1}{2}\rho^b(x)\tau^b) + i\frac{1}{2}g_F L\tau^a F^a_\mu - \frac{1}{4}g_F L\tau^a F^a_\mu\rho^b\tau^b\end{array}$$

The right of (A.8) is

$$\begin{array}{l}(\partial_\mu L + i\frac{1}{2}g_F L\tau^a F^a_\mu)(1+i\frac{1}{2}\rho^b(x)\tau^b) \\ = \partial_\mu L(1+i\frac{1}{2}\rho^b(x)\tau^b) + i\frac{1}{2}g_F L\tau^a F^a_\mu - \frac{1}{4}g_F L\tau^a F^a_\mu\rho^b\tau^b\end{array}$$

Thus (A.8) can be established.
So, (A.4) is invariant under $SU(2)_L \otimes U(1)_Y \otimes SU(2)_N$.

B. The cross section of neutrino-neutrino interaction σ

We assign neutrino energy $E_v = 1GeV$,and some known paramters as below

$$\begin{array}{l}m_{v_e} \approx 1eV = 10^{-9}GeV \\ M_Z \approx 90GeV \\ g_Z^2 = \frac{M_Z^2}{4\sqrt{2}}G_F \\ G_F = 1 \times 10^{-5}/(GeV)^2\end{array}$$

Taking above values into the cross section formula, we will get

$$\sigma \approx \frac{g_z^4 \times 2m_{v_\alpha} E_{v_\alpha}}{\pi M_z^4} = \frac{\frac{M_Z^4}{32} \times G_F^2 \times 2m_{v_e} E_{v_e}}{\pi M_z^4} = \frac{10^{-19}}{16\pi}(GeV)^{-2} \qquad (B.1)$$

Using the conversion of natural units, we can get

$$\begin{aligned} &1cm = 5 \times 10^{13} GeV^{-1} \\ &\to GeV^{-1} = 0.2 \times 10^{-13} cm \\ &\to GeV^{-2} = 0.04 \times 10^{-26} cm^2 \end{aligned}$$

Thus

$$\sigma \approx \frac{10^{-19}}{16\pi}(GeV)^{-2} = \frac{1}{4\pi} \times 10^{-47} cm^2$$

Because the cross section formula (B.1) is obtained with the convention that the impact parameter $b \sim 0$, we adopt the unit cm^2.

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