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High Energy Approximation for Monte Carlo Event generator

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Analysis of the nuclear physics experiment often requires Monte Carlo simulation of the detector setup. Such kind of simulations often requires a quite advanced model of nuclear interaction. In this work we describe application of Monte Carlo simulation in the nuclear physic experiment, and discuss on example of elastic scattering how to apply well known High Energy Approximation (or Glauber model) to Monte Carlo simulation of the experimental setup.

Keywords: Monte-Carlo method, high-energy approximation, Glauber model, elastic scattering, cross section calculations

Introduction

The Monte Carlo (MC) method [1-3] is widely used for simulations of various processes in different areas of science. In the present study, this method serves for verification of certain theoretical model of nuclear reaction. The primary events are generated on the basis of the theoretical model and the resulting data of these simulations are compared to the experimental data, thus verifying the model.

The general scheme of such analysis is shown in scheme (Figure 1).

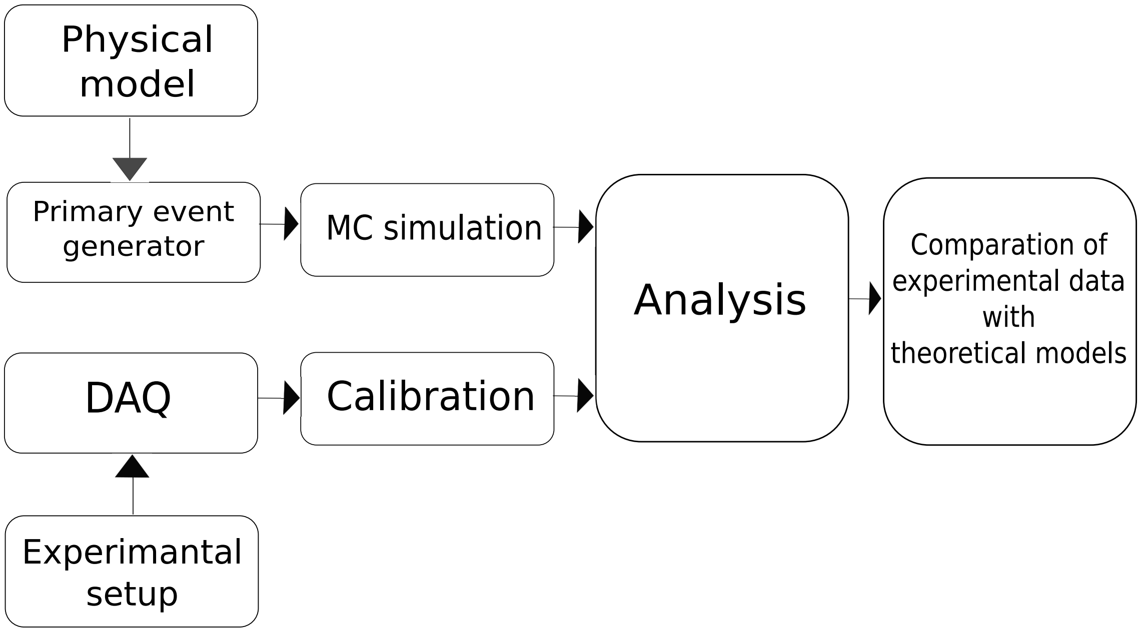

Figure 1. Analysis of experimental data based on MC simulations.

The first step, implemented by program (or subprogram), is called Primary Event Generator.

As one can see in Figure 1 Primary Event Generator connects MC simulation with a theoretical model. Thus, the Monte Carlo simulation can be divided into three steps:

1. Primary event generation;

2. Particle transport through virtual detector setup;

3. Evaluation of the detector response.

A good example of how such scheme is realized for nuclear data analysis in most of the leading scientific nuclear laboratories in Europe is the GEANT code. This comprehensive tool, used for simulating the interactions of various particles with different materials under various conditions, is developed and maintained by the European Organization for Nuclear Research (CERN). The detailed description one can find in Refs. [4, 5]. The input data for GEANT are the momenta of the particles, called primary events should be generated independent way in the frames of the physical model of a certain nuclear reaction. Note, that the primary event generator should satisfy some common demands:

1. Relative simplicity of theoretical model for the fastest event generation;

2. Use of the same dispersion relation as in the particle transport codes (for example GEANT);

3. To make the tool easy-to-use for experimentalists.

A general aim of the work is to create such tool for primary event generation, that will serve for making comparison of the model calculations with experimental data.

In the present paper we show how to adopt theoretical model for primary event generator.

As a first approach we prepare the primary events for the reactions of elastic scattering of light fragmented (few-body) nuclei [6-8]on the basis of Glauber-type models, which allows for relatively simple and fast calculations.

Note, that the Glauber-type models needs modification in order to regard for the energy conservation law. For this we introduce the term accounting for the phase volume and transit from the eikonal transferred momentum to realistic transferred momentum. The calculations of reactions with few-body nuclear systems demand performing the multi-dimensional integration. And the most efficient way for this is the MC integration. Thus, the MC simulation is used both at the primary event generation step and the step of beam transport through the experimental setup.

Cross section and reaction rate

Cross section of the elastic scattering with arbitrary number of products in final state can be written as

$$\sigma \propto \int T^2 \mathrm{d}V^n, \tag{1}$$

where

$$\mathrm{d}V^n = \delta^4 \left(P_I - P_F\right) \prod_{i=1}^{n} \frac{\mathrm{d}^3\mathbf{p}_i}{E_i}$$

is the n -body phase space, responsible for the energy and momenta conservation law.

This representation of the cross section is suitable for the primary event generator because the energy and momenta conservation law for each participant is factorized.

As shown in [5] the Eq. (1) can be written as

$$\int T^2 \mathrm{d}V^n = \int T^2 R^n(\varepsilon_1, \ldots, \varepsilon_k) \prod_{i=1}^{k} \mathrm{d}\varepsilon_i, \quad k = 3n-4,$$

where ε_i are independent kinematiclal varaibles.

For the case of the elastic scattering and binary reaction one obtains

$$R^2(\mathbf{k}, E) = \frac{k}{E} \mathrm{d}^2\hat{\mathbf{k}}, \tag{2}$$

where $\mathbf{k}$ is the momenta in center-of-mass system (c.m.s.), E — the system total energy. Thus for the binary reaction, the phase space is a sphere.

For the non-relativistic limit, E reduces to sum of the reaction product masses and (2) can be writen as

$$R^2(\mathbf{k}) = \frac{k}{m_1 + m_2} \mathrm{d}^2\hat{\mathbf{k}}, \tag{3}$$

where m_1 and m_2 are the masses of the reaction products.

The elastic scattering T-matrix elements

The total cross section of the elastic scattering can be writen as

$$\sigma_{\mathrm{el}} = \int \left| F\left(\mathbf{k}', \mathbf{k}\right) \right|^2 \mathrm{d}^2\hat{\mathbf{k}}', \tag{4}$$

where

$$F\left(\mathbf{k}', \mathbf{k}\right) \equiv F\left(\mathbf{q}\right)$$

is the scattering amplitude, $\mathbf{k}$, $\mathbf{k}'$ are the c.m.s. momenta in initial and final states, $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ are the transferred momenta.

Comparing equations (1), (3), and (4) one can see that the T-matrix element is connected to the scattering amplidude in a quite simple way:

$$T_{\mathrm{el.}} = \sqrt{\frac{1}{k'}} F. \tag{5}$$

Also note that the momentum $\mathbf{k}'$ integration over the volume of the sphere is equivalent to the momentum $\mathbf{q}$ integration over the squere of the circle:

$$\mathrm{d}^2\hat{\mathbf{k}}' = \sin\theta \mathrm{d}\theta \mathrm{d}\phi = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\mathrm{d}\theta\mathrm{d}\phi = \frac{1}{k^2} q\mathrm{d}q\mathrm{d}\phi,$$

$$\mathrm{d}^2\hat{\mathbf{k}}' = \frac{\mathrm{d}^2\mathbf{q}}{k^2}.$$

Elastic scattering in the frame of the Glauber model

The Glauber model [9, 10] is a well developed approach of nuclear reaction theory.

In the frame of the Glauber model, the scattering amplitude of a fragment is defined as

$$F(\tilde{\mathbf{q}}) = \frac{k}{i} \int f(\tilde{\mathbf{q}}, \mathbf{b}) \mathrm{d}^2\mathbf{b}, \tag{6}$$

where $\tilde{\mathbf{q}}$ is eikonal transfered momentum; $\mathbf{b}$ is the impact parameter of the fragment and

$$f(\tilde{\mathbf{q}}, \mathbf{b}) = \frac{e^{i\tilde{\mathbf{q}}\mathbf{b}}}{2\pi} \left[\exp\left(iX(\mathbf{b})\right) - 1\right],$$

here X is optical phase. The integration in (6) is over the plane orthogonal to $\mathbf{k}$. Using the integral representation of the Bessel function J_0:

$$J_0(\chi) = \frac{1}{2\pi} \int_0^{2\pi} e^{i\chi\cos\phi} \mathrm{d}\phi$$

with the case of the symmetric potential eq. (6) can be integrated over the impact parameter angle

$$F(\tilde{q}) = \frac{k}{i} \int f(\tilde{q}, b) b\mathrm{d}b,$$

where

$$f(\tilde{q}, b) = J_0\left(\tilde{q}b\right) \left[\exp\left(iX(\mathbf{b})\right) - 1\right].$$

and in this case the scattering amplitude doesn't depend on direction $\tilde{\mathbf{q}}$. The optical theorem in the Glauber is written as

$$\sigma = \frac{4\pi}{k}\Im F(0) = \int |F(\tilde{\mathbf{q}})|^2 \frac{d^2\tilde{\mathbf{q}}}{k^2}, \tag{7}$$

and it can be seen that the expression for the imaginary part $\Im$ of the forward scattering amplitude $F(\mathbf{k},\mathbf{k}') = F(0)$ in the high-energy approximation satisfies the optical theorem, unlike, for example, the Born approximation.

One of key features of the model is that instead of energy conservation, we get longitudinal momentum conservation.

In terms of phase space this means that the momentum vector $\mathbf{k}'$ moves not on the sphere but on the plane. So the Glauber model has drastically different phase space.

In terms of the transferred momenta this difference leads to changes in the region of integration over $d^2\mathbf{q}$ from the area restricted by $2k$ circle to the whole plane.

Therefore energy conservation law can be formally secured in the frame of the Glauber model by claming relation between q and $\tilde{q}$ as follow

$$\tilde{q} = q = 2k\sin(\theta/2), \tag{8}$$

$$\tilde{q} = k\tan\theta,\ q = 2k\sin(\theta/2), \tag{9}$$

where θ is the scattering angle in c.m.s. In the Eq. (8) (suggested in [9]) unitarity is broken but the phase space is preserved. In the Eq. (9) the unitarity is preserved but phase space is significantly modified.

However the relation (8) should be preferably considered for events generation.

Monte Carlo calculation

Finaly we discuss an application of the Monte Carlo method for cross section calculations and the event generation.

In the case of potentials with finite range R, for the evaluation of the integrals in (6) it is convenient to replace integration over the impact parameter by integration over some dimensionless parameter

$$B = b^2/R^2, \tag{10}$$

thus

$$F(\tilde{q}) = R^2\frac{k}{2i}\int_0^1 f(\tilde{q}, R\sqrt{B})dB. \tag{11}$$

For integration over $\tilde{q}$ we apply the substitution

$$C = 1 - \cos\theta, \quad \tilde{q} = q = k\sqrt{2C}. \tag{12}$$

Now we can write expression for the cross section

$$\sigma = 2\pi R^2 \Im \left[i \int_0^1 \left(1 - \exp\left(iX(R\sqrt{B})\right)\right) \mathrm{d}B \right], \tag{13}$$

$$\sigma' = \frac{\pi}{2} R^4 k^2 \int_0^2 \mathrm{d}C \int_0^1 \mathrm{d}B \int_0^1 \mathrm{d}B' f(k\sqrt{2C}, R\sqrt{B}) f^*(k\sqrt{2C}, R\sqrt{B'}). \tag{14}$$

Equation (14) represents the transformed cross-section (13), where integration is carried out over the new variables C, B and B', related to the scattering angle θ and the transferred momentum q, respectively.

Eqs. (13) and (14) can be calculated numerically

$$\sigma = 2\pi R^2 \frac{1}{N} \Re \left[\sum_{i=1}^{N} \left(1 - \exp\left(iX(R\sqrt{B_i})\right)\right) \right], \tag{15}$$

$$\sigma' = \frac{\pi}{4} R^4 k^2 \sum_{i=1}^{N} f(k\sqrt{2C_i}, R\sqrt{B_i}) f^*(k\sqrt{2C_i}, R\sqrt{B_i'}). \tag{16}$$

Here B and B' are the random numbers uniformly distributed in range $[0,1]$; C — in range $[0,2]$.

The comparison function for generating events with momentum distibution $\hat{\mathbf{k}}_i'$ corresponging to (14) using von Neumann rejection can be evaluated as

$$\mathrm{d}\sigma(\mathbf{k}) = \frac{\pi}{2} R^4 k^2 \Re \left(f(k\sqrt{2C}, R\sqrt{B}) f^*(k\sqrt{2C}, R\sqrt{B'}) \right). \tag{17}$$

Numerical example

We make test calculation for the case of the square well potential with width R and depth V. The optical phase in this case

$$X(B) = -2\alpha\sqrt{1-B}, \quad \alpha = \frac{VR}{\hbar v},$$

where v is the c.m.s velocity. And the total cross section can be calculated analytically as

$$\sigma = \pi R^2 \left(2 + \frac{1}{\alpha^2} - \frac{2}{\alpha} \left[\frac{\cos(2\alpha)}{2\alpha} + \sin(2\alpha) \right] \right). \tag{18}$$

The calculation parameters and results presented in Table 1. With reasonable numbers of events the integrals (15) and (16) converge.

As expected substitution (8) significantly breaks optical theorem, where σ_a was calculated according to Eq. (18), σ_n - according to Eq. (15), $\Re\sigma'$ and $\Im\sigma'$ were calculated according to the Eq. (16).

Table 1.
Calculation parameters and results.

N	10^6	
v	0.25	
R	5	fm
V	3	MeV
k	1.23	fm^{-1}
σ_a	14.23	fm^2
σ_n	14.23	fm^2
$\Re\sigma'$	3.593	fm^2
$\Im\sigma'$	$-4.4 \cdot 10^{-4}$	fm^2

Figures 2 and 3 show distribution of generated event by scattering angle and distribution of generated event by transferred momentum, respectively.

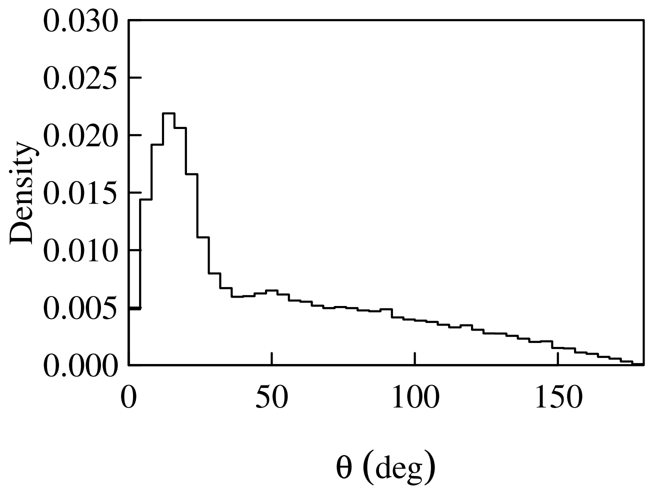

Figure 2. Event distribution by scattering angle in center-of-mass system.

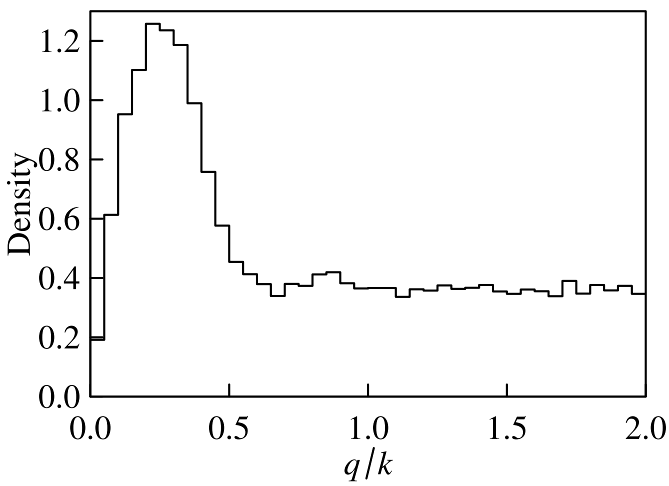

Figure 3. Event distribution by transfered momentum.

Results and conclusion

We discussed the simple case of elastic scattering and provide expression (17) for using in primary event generator.

In the paper we considered the example of the phase space modification. It was found that in the Glauber-type model the regard for the energy- and momentum conservation law leads to transfer from the eikonal transferred momentum to the "realistic" transferred momentum. As the result of the angular distribution calculations, we got the elastic scattering into back-angles.

Note that for generation of the events we don't need calculations of the scattering amplitude for each event, the cross section is reduced to the triple integration over two impact parameters and transferred momentum and the von Neumann rejection method can be applied for the three-dimensional distribution, that provides convergence of the method. The question whether this procedure can also be expanded and adapted for other cases of more complex reactions, for example, inelastic scattering when the overlap of different states should be calculated, remains open.

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