

# Microscopic analysis of the $^{12,14}\text{Be}$ scattering on $^{12}\text{C}$ and protons

E.V. Zemlyanaya<sup>\*1</sup>, V.K. Lukyanov<sup>1</sup>, K.V. Lukyanov<sup>1</sup>,  
D. Kadrev<sup>2</sup>, M.K. Gaidarov<sup>2</sup>, A.N. Antonov<sup>2</sup>, K. Spasova<sup>2</sup>

<sup>1</sup>Joint Institute for Nuclear Research, Dubna, Russia

<sup>2</sup>Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Bulgaria

E-mail: elena@jinr.ru

DOI: 10.29317/ejpfm.2019030107

Received: 15.03.2019 - after revision

Differential cross sections of elastic scattering of  $^{12,14}\text{Be}$  on  $^{12}\text{C}$  and protons are analyzed within the microscopic model of the optical potential (OP). The microscopic OP consists of the double folding real part and the imaginary part which is constructed using the high energy approximation theory. The OP depends on the nuclear density distributions of  $^{12,14}\text{Be}$  and thus, their microscopic models are tested in our study.

**Keywords:** exotic nuclei, microscopic optical potential, differential cross sections.

## Introduction

The present study aims to analyze the elastic scattering of the neutron-rich isotopes  $^{12,14}\text{Be}$  on  $^{12}\text{C}$  and proton targets by using of microscopically calculated optical potential (OP), where the nucleon density distributions of the exotic nuclei  $^{12,14}\text{Be}$  are being of the main physical interest. The experimental data of  $^{12,14}\text{Be} + ^{12}\text{C}$  scattering have been published in [1] and interpreted with the help of phenomenological OP [1, 2]. However, a reasonable agreement with the data was obtained by fitting of more than 10 phenomenological parameters. Moreover, the values of these parameters in [1, 2] occurred to be very different. Thus, the problem to explain the experimental data on the basis of a realistic theoretical approach is still open.

The elastic  $^{12,14}\text{Be}+p$  scattering cross sections at 700 MeV/nucleon have been measured and explained in [11] within the Glauber theory by using the phenomenological  $^{12,14}\text{Be}$  density distributions in the form of the symmetrized Fermi function (SF).

In our study, the hybrid microscopic OP is used [3, 4], where the real part of OP is constructed within the double folding model (DF) [5] accounting for the antisymmetrization of the whole wave function. As to the imaginary part of OP, it is calculated on the basis of the high-energy approximation (HEA) [6].

## Theoretical framework

The DF OP consists of the direct and exchange terms,  $V^D$  and  $V^{EX}$  [3-5]:

$$V^{DF}(r) = V^D(r) + V^{EX}(r). \quad (1)$$

Both potentials are composed from the isoscalar and isovector terms and the former one is determined as follows:

$$V^D(r) = \int d^3r_p d^3r_t \rho_p(\vec{r}_p) \rho_t(\vec{r}_t) v_{NN}^D(s), \quad (2)$$

$$V^{EX}(r) = \int d^3r_p d^3r_t \rho_p(\vec{r}_p, \vec{r}_p + s) \rho_t(\vec{r}_t, \vec{r}_t - s) v_{NN}^{EX}(s) \exp\left[\frac{i\vec{K}(r)s}{M}\right]. \quad (3)$$

Here  $\vec{s} = \vec{r} + \vec{r}_t - \vec{r}_p$  is the vector between two nucleons, one of which belongs to the projectile and another one to the target nucleus.  $\rho_{p,t}$  projectile and target densities,  $\vec{K}(r)$  - local momentum of the nucleus-nucleus relative motion,  $v_{NN}^{D,EX}$  - effective Paris NN CDM3Y potentials parameterized in [7]. The isovector potential is determined by the same formulas (2, 3) but  $\rho_i$  ( $i = t, p$ ) should be exchanged by  $\delta\rho_i$ , the difference between proton and neutron densities for every  $i$ -nucleus.

At comparably high energies, the NN potential is expressed through its explicit form [6]. In this framework, the HEA OP is determined as follows [3]:

$$U_{opt}^H(r) = -\frac{E}{k} \bar{\sigma}_N (i + \bar{\alpha}_N) \frac{1}{(2\pi)^3} \int e^{-i\vec{q}\vec{r}} \rho_p(q) \rho_t(q) f_N(q) d^3q. \quad (4)$$

Here  $\bar{\sigma}_N$  is the isospin averaged NN total cross section,  $\bar{\alpha}_N$  is the ratio of real to imaginary part of the forward nucleon-nucleon amplitude, and  $f_N(q) = \exp(-\beta_N q^2/2)$ , where  $\beta_N$  is the slope parameter.

The hybrid form of the microscopic nucleus-nucleus OP is:

$$U(r) = N_R V^{DF}(r) + iN_I W^H(r) \quad (5)$$

where  $N_R$  and  $N_I$  are the renormalization factors of the real and imaginary OPs which are adjusted to the experimental data. In the case of the known densities of interacting nuclei, there are no more parameters to be fitted.

The standard DWUCK4 [8] code is used for calculation of the cross sections. The Coulomb potential is taken in the standard form of the uniformly charged sphere of the radius  $R_C$ .

We use the following densities of  $^{12,14}\text{Be}$ :

- Microscopic density calculated within the generator coordinate method (GCM) [9]. In this framework, the  $^{14}\text{Be}$  nucleus is considered as a three-cluster nucleus, involving several  $^{12}\text{Be} + n + n$  configurations. The  $^{12}\text{Be}$  core nucleus is described in the harmonic oscillator model with all possible configurations in the  $p$  shell.

- In the variational Monte Carlo model (VMC) [10], the proton and neutron densities are computed with the so called AV18+UX Hamiltonian, in which the Argonne v18 two-nucleon and Urbana X three-nucleon potentials are used.

- Phenomenological density in the form of the symmetrized Fermi function (SF):

$$\rho_{SF}(r) = \frac{A}{\frac{4}{3}\pi R^3} \left[ 1 + \left( \pi \frac{a}{R} \right)^2 \right]^{-1} \times \frac{\sinh\left(\frac{R}{a}\right)}{\cosh\left(\frac{r}{a}\right) + \cosh\left(\frac{R}{a}\right)}. \quad (6)$$

Here the parameters, radius  $R$  and diffuseness  $a$ , were established in [11] by fitting (within the Glauber approach) to the data of  $^{12,14}\text{Be} + p$  elastic scattering at 700 MeV/nucleon:  $R=1.37$  fm,  $a=0.67$  fm for  $^{12}\text{Be}$  and  $R=0.99$  fm,  $a=0.84$  fm for  $^{14}\text{Be}$ .

The  $^{12}\text{C}$  density is taken in the modified SF form:

$$\rho(r) = \rho_{SF}(r) + \rho_{SF}^{(1)}(r), \quad (7)$$

where the  $\rho_{SF}$  is determined by Eq. (6) with radius  $R=2.275$  fm and diffuseness  $a=0.393$  fm and the surface term  $\rho^{(1)}$  is calculated via the 1st derivative of  $\rho_{SF}$ . The parameters of this density were obtained in [12] by fitting to the  $eA$  scattering data.

## Results

It was suggested in [1] and [2] that the experimental data should be considered as quasielastic scattering, i.e., the contribution of inelastic channels related to excitations of the low-lying collective states of a nucleus, should be accounted for. Within our microscopic approach, the inelastic OP was calculated via the derivative of the microscopic OPs:  $U_{inel} = -\tilde{R} \cdot dU/dr$ , where  $U$  is microscopic OP in the form (5),  $\tilde{R}$  is the potential radius (we put  $\tilde{R}=4.25$  fm as in [1]). At this stage, we only accounted for excitation of the  $2^+$  state ( $E_{2^+}=4.436$  MeV). In this case, there is one more fitting parameter apart from the factors  $N_R$  and  $N_I$ , the deformation parameter  $\beta_2$ . The results are shown in Figure 1. The calculations have been performed with different densities of  $^{12,14}\text{Be}$ . The values of the parameters  $N_R$ ,  $N_I$ , and  $\beta_2$  are given in Table 1. It is seen that the SF density provides better agreement of theoretical curve (solid line) with the experimental data on ( $^{12}\text{Be} + ^{12}\text{C}$ ) scattering in comparison with the GCM and VMC model densities of  $^{12}\text{Be}$ . In the case of ( $^{14}\text{Be} + ^{12}\text{C}$ ) scattering, a noticeable discrepancies with experimental data are observed in both cases of SF and GCM densities. One expects that accounting for excitation of the  $3^-$  state in the inelastic channel can provide an agreement with the experimental data to be comparable with results in [1, 2] on the basis of phenomenological approach.

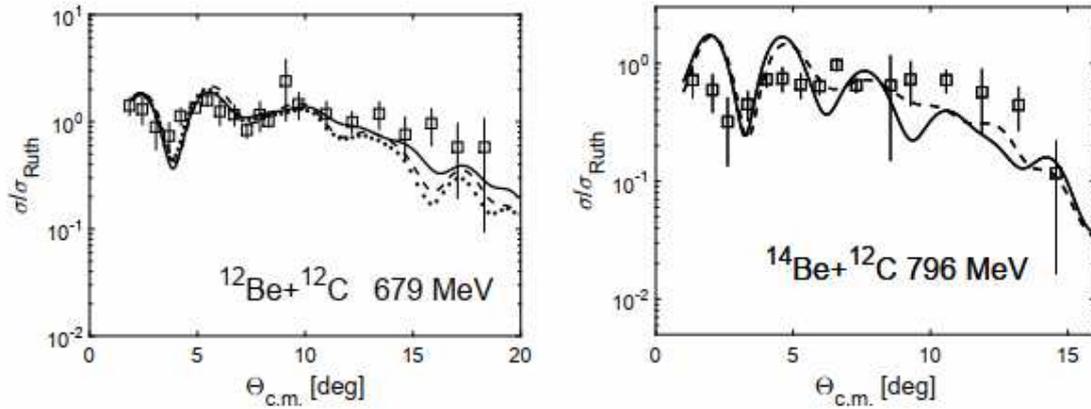


Figure 1. The differential cross sections of the  $^{12,14}\text{Be} + ^{12}\text{C}$  quasielastic scattering with accounting for the  $2^+$  inelastic channel. Solid, dashed, and dotted lines correspond, respectively, to the SF, GCM, and VMC densities of  $^{12,14}\text{Be}$  used in calculations of OP.

Table 1.

Results of measurements with a spherical albedo system and calculations by the spectrum.

Nucleus	Density	Figure 1			Figure 2		
		$N_R$	$N_I$	$\beta_2$	$N_R$	$N_I$	$\chi^2/N$
$^{12}\text{Be}$	GCM	0.42	1.09	0.66	1.51	0.93	12.63
	SF	0.57	0.91	0.78	0.93	1.17	0.99
	VMC	0.42	1.10	0.59	1.41	1.01	2.39
$^{14}\text{Be}$	GCM	0.36	1.32	0.42	1.08	0.92	12.43
	SF	0.43	0.62	0.48	0.77	1.10	1.10

The results of calculation of differential cross sections of the ( $^{12,14}\text{Be}+p$ ) elastic scattering are presented in Figure 2. Here, the HEA OP [Eq. (4)] was used with parameters  $\bar{\alpha}_n, \bar{\beta}, \bar{\sigma}_n$  as they done in [11]. The cross sections are calculated by means of numerical solution of the respective relativistic equation, see [13] for details. The calculations can reasonably reproduce the data of  $^{12,14}\text{Be}+p$  elastic scattering even in case of  $N_R=N_I=1$ . Having in mind that the cross sections in our study and in [11] are calculated on different backgrounds, we have fitted parameters  $N_R$  and  $N_I$  of the microscopic OP and improved the agreement of calculations with experimental data, see Figure 2. The values of the best fit parameters  $N_R, N_I$ , and  $\chi^2/N$ , where  $N$  stands for the number of the experimental points, are listed in the Table 1. One sees that the calculations with SF and VMC densities well reproduce the experimental data, while the GCM model does not provide so good agreement at  $\theta > 5^\circ$ .

## Summary

The differential cross sections of the ( $^{12,14}\text{Be} + ^{12}\text{C}$ ) elastic scattering at energy 56 MeV/nucleon and of the ( $^{12,14}\text{Be}+p$ ) at energy 700 MeV/nucleon have been analyzed within the hybrid model of the microscopic OP. Three models of the  $^{12,14}\text{Be}$  density distributions are tested. It is shown that the inclusion of both

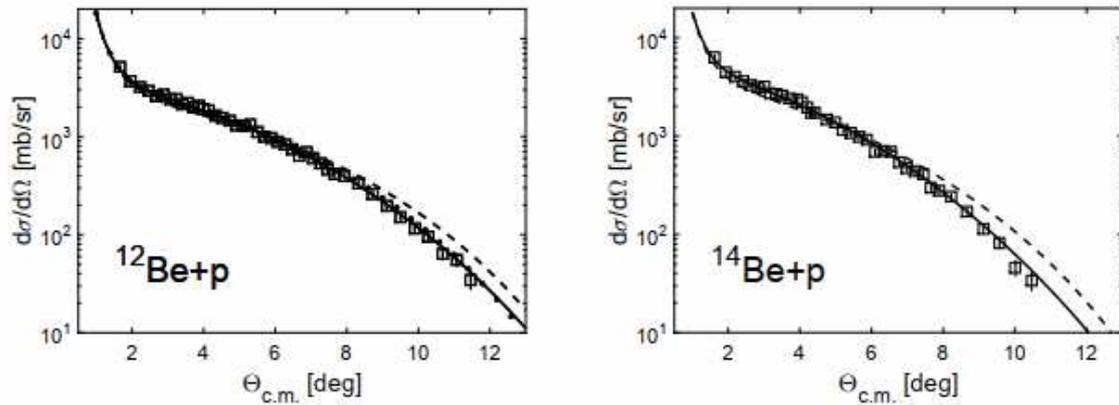


Figure 2. The differential cross sections of the  $^{12,14}\text{Be}+p$  elastic scattering at 700 MeV/nucleon. Calculations are performed with the modified SF-density of  $^{12}\text{C}$  (7) and the different densities of  $^{12,14}\text{Be}$ . Solid, dashed and dotted lines correspond, respectively, the SF, GCM, and VMC densities of  $^{12,14}\text{Be}$ .

elastic and inelastic channels in the calculations allows to explain the experimental data on the ( $^{12,14}\text{Be}+^{12}\text{C}$ ) elastic scattering at energy of 56 MeV/nucleon with the given resolution. Also, the microscopic OPs for the ( $^{12,14}\text{Be}+p$ ) scattering calculated with the VMC and SF densities of  $^{12,14}\text{Be}$  provide an agreement with the experimental data at energy about 700 MeV/nucleon.

## Acknowledgements

This work is partly supported by the grant of the "JINR-Bulgaria" Program, by the DFNI-T02/19 and DNTS/Russia 01/3 grants of the Bulgarian Science Foundation and by the Russian Foundation for Basic Research (grants No. 17-52-18057-bolg-A, 17-01-00661A). This financial support is gratefully acknowledged.

## References

- [1] M. Zahar et al., Phys. Rev. C **49** (1994) 1540.
- [2] M.C. Mermaz, Phys. Rev. C **50** (1994) 2620.
- [3] V.K. Lukyanov et al., Phys. At. Nucl. **69** (2006) 240.
- [4] K.V. Lukyanov, JINR Communication R11-2007-38 (2007).
- [5] D.T. Khoa, G.R. Satchler, Nucl. Phys. A **668** (2000) 3.
- [6] R.J. Glauber, Lectures in Theor. Physics (New-York: Interscience, 1959) 315 p.
- [7] N. Anataraman et al., Nucl. Rev. A **398** (1983) 269.
- [8] P.D. Kunz, E. Rost, Computational Nuclear Physics **2** (1993) 88.
- [9] P. Descouvemont, Phys. Rev. C **52** (1995) 704.
- [10] S.C. Pieper, private communication.
- [11] S. Ilieva et al., Nucl. Phys. A **875** (2012) 8.
- [12] V.V. Burov et al., Phys. At. Nucl. **61** (1998) 525.
- [13] V.K. Lukyanov et al., Phys. At. Nucl. **78** (2015) 142.