

# Fission of super-heavy nuclei: Fragment mass distributions and their dependence on excitation energy

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The mass and total kinetic energy distributions of the fission fragments in the fission of even-even isotopes of superheavy elements from Hs ( $Z=108$ ) to Og ( $Z=118$ ) are estimated using a pre-scission point model. We restrict to nuclei for which spontaneous fission has been experimentally observed. The potential energy surfaces are calculated with Strutinsky's shell correction procedure. The parametrization of the nuclear shapes is based on Cassini ovals. For the just before scission configuration we fix  $\alpha = 0.98$ , what corresponds to  $r_{neck} \approx 2$  fm, and take into account another four deformation parameters:  $\alpha_1, \alpha_2, \alpha_4, \alpha_6$ . The fragment-mass distributions are estimated supposing they are due to thermal fluctuations in the mass asymmetry degree of freedom just before scission. The influence of the excitation energy of the fissioning system on these distributions is studied. The distributions of the total kinetic energy (TKE) of the fragments are also calculated (in the point-charge approximation). In Hs, Ds and Cn isotopes a transition from symmetric to asymmetric fission is predicted with increasing neutron number  $N$  (at  $N \approx 168$ ). Super-symmetric fission occurs at  $N \approx 160$ . When the excitation energy increases from 0 to 30 MeV, the peaks (one or two) of the mass distributions become only slightly wider. The first two moments of the TKE distributions are displayed as a function of the mass number  $A$  of the fissioning nucleus. A slow decrease of the average energy and a minimum of the width (at  $N \approx 162$ ) is found.

**Keywords:** SHE, nuclear fission, Hs, Ds, Cn, Fl, Lv, Og isotopes, symmetric vs asymmetric fission, excitation energy dependence.

## Introduction

Projects to measure fission fragment properties for the heaviest elements ( $Z > 106$ ) are underway at several heavy-ion facilities around the world. The SHE Factory of JINR-Dubna [1], which will produce its first intense beam in 2019, the HIAF+CUBE of ANU-Canberra [2] which will produce such nuclei through quasifission reactions and the tandem accelerator of the JAEA-Tokai which will use fusion-fission reactions [3] are just few examples. In this context, theoretical calculations are very useful. On one side they can improve the design of such experiments and on the other side they provide predictions to compare with data allowing to test various theoretical assumptions.

In the present study, an improved version of the scission point model [4] that has confirmed its ability to describe the mass and kinetic energy distributions of the fission fragments from the spontaneous fission of the heaviest actinides for which such distributions have been measured [5], is used. More recently, the same model was applied to long series of isotopes with atomic number  $Z$  from 110 to 126 in order to study general trends [6]. Thus, if one decreases the number of neutrons  $N$  and includes the octupole deformation  $\alpha_3$ , a transition from asymmetric to symmetric mass division takes place in Fl, Lv, Og and  $Z=126$  isotopes. It is the mirror of the behaviour in Fm, No, Rf and Sg isotopes where the same transition occurs with increasing  $N$ . In this way the fragment mass systematics of the SHE and of the heavy actinides join smoothly together as shown in Figure 1. It is a test of the reliability of the present approach. It is interesting to notice that each series of isotopes has its narrowest symmetric mass distribution with a full width at half maximum between 5 and 8 amu; hence extremely small. To distinguish it from the regular symmetric fission, we call this type of fission "super-symmetric".

The influence of the double magic  $^{132}\text{Sn}$  is clearly seen by the almost constant mass ( $\approx 136$ ) of the heavy fragment in actinides and of the light fragment in superheavies. The masses of the complementary fragment lie on a straight line, simply reflecting the conservation of the total mass number  $A$ .

However, the fission of many of these nuclei will never be observed and their study is only academic. Here we concentrate on nuclei with  $Z > 106$  for which spontaneous fission has been already detected although the statistics was not enough to build distributions. As mentioned earlier, some of these nuclei will soon be remeasured in better conditions. Our goal is to anticipate such experiments through a detailed theoretical description of their fission fragments' properties.

Although the total kinetic energy of the fragments is also calculated, the accent is put on their masses. Is the mass division expected to be symmetric or asymmetric? How does the excitation energy of the fissioning nucleus influences this mass division? To answer such questions, we use a pre-scission point model and therefore calculate the potential energy surface (PES) of deformation, with Strutinsky's macroscopic-microscopic method [7], for a fissioning nucleus just before its separation into two fragments. These last mono-nuclear shapes are described by generalized Cassini ovals with up to five deformation parameters  $\{\alpha_i\}$ . The corresponding collective degrees of freedom (normal to the fission direction) are supposed to be in statistical equilibrium. We therefore estimate

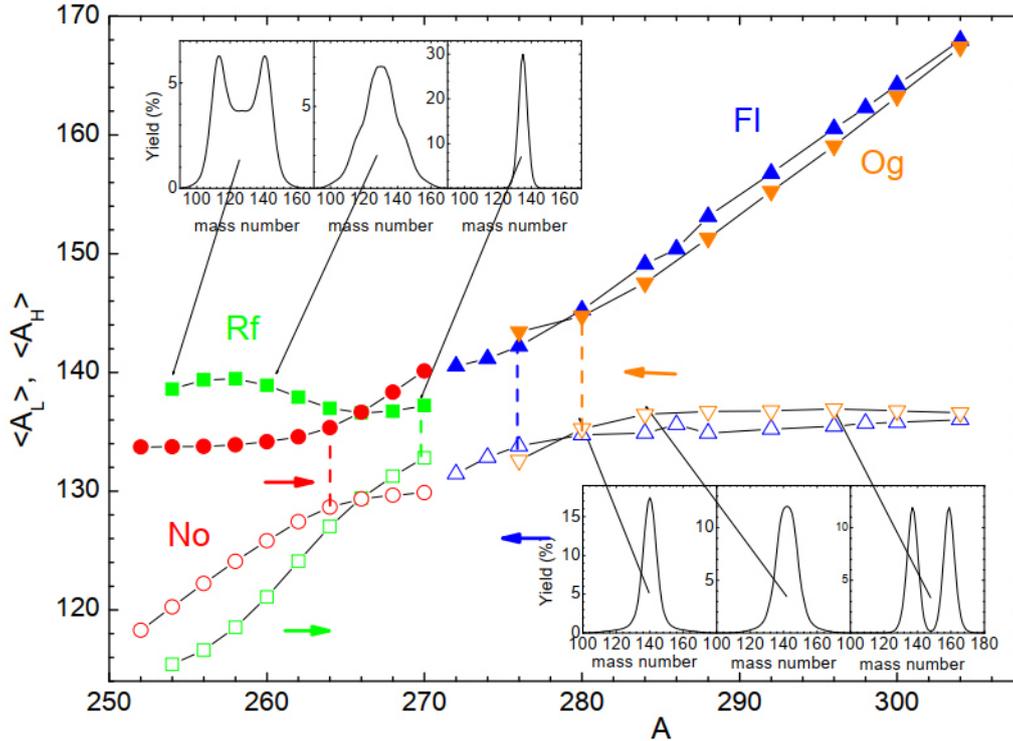


Figure 1. Average masses of the light and heavy fragments for isotopes of No, Rf, Fl and Og as a function of the mass number  $A$  of the fissioning nucleus. The start points of the arrows mark the transition from asymmetric to symmetric mass division and dashed verticals mark the narrowest symmetric mass distribution in each series of isotopes.

the mass and TKE distributions using Boltzman factors for the probability to populate the points  $\{\alpha_i\}$  on the PES. As for the excitation energy dependence of PES, the Strutinsky shell correction in excited nucleus is calculated keeping the same entropy (not temperature) in the original and averaged quantities, so that it decreases monotonically, more or less exponentially, with excitation energy.

The computational approach is explained in Sec. II. In Sec. III the predicted fission fragment mass and TKE distributions are presented for the Hs, Ds, Cn, Fl, Lv and Og isotopes for which spontaneous fission has been detected. The effect of the excitation energy on the mass distributions is estimated. A summary and conclusions can be found in Sec. IV.

## Computational details

Our model is a "just-before scission" model [6] that uses generalized Cassinian ovals

$$R(x) = R_0 \left[ 1 + \sum_n \alpha_n P_n(x) \right], \quad (1)$$

to describe the ensemble of nuclear shapes involved.  $R_0$  is the radius of the spherical nucleus,  $P_n(x)$  are Legendre polynomials and  $\alpha_n$  are the shape (deformation) parameters. These shapes have in common a parameter  $\alpha$  chosen such that at  $\alpha = 1.0$  the neck radius is equal to zero irrespective of the values of  $\alpha_n$ . For the just before scission configuration we fix  $\alpha = 0.98$  [8], and take into account another four

deformation parameters  $\alpha_1, \alpha_3, \alpha_4, \alpha_6$ . With the shape parametrization (1) we calculate the potential energy of deformation using the microscopic - macroscopic approach [7]:

$$E_{def} = E_{def}^{LD} + \delta E, \quad (2)$$

where  $E_{def}^{LD}$  is the macroscopic liquid-drop energy including surface and Coulomb energies and  $\delta E$  contains the microscopic shell and pairing corrections.

Supposing statistical equilibrium for the collective degrees of freedom normal to the fission direction [9], the distribution of these probabilities is

$$P(\alpha_1, \alpha_3, \alpha_4, \alpha_6) \propto e^{-E_{def}(\alpha_1, \alpha_3, \alpha_4, \alpha_6)/T_{coll}}, \quad (3)$$

Projecting  $P(\alpha_1, \alpha_3, \alpha_4, \alpha_6)$  on the fixed value of mass asymmetry  $\eta = (A_H - A_L)/A$  one obtains the fission fragment mass distribution  $Y(\eta)$ ,

$$Y(\eta) = \frac{\sum_{ijk} P(\alpha_1(\eta), \alpha_{3i}, \alpha_{4j}, \alpha_{6k})}{\int d\eta \sum_{ijk} P(\alpha_1(\eta), \alpha_{3i}, \alpha_{4j}, \alpha_{6k})}, \quad (4)$$

$T_{coll}$  is an unknown parameter that controls the overall width of the distribution. In a sense it takes partially into account the dynamics.

In a similar way one obtains the fission fragment total kinetic energy (TKE) distribution. For each point  $\alpha_1, \alpha_3, \alpha_4, \alpha_6$  one calculates the Coulomb interaction of the fragments

$$E_{coul}^{int} = e^2 Z_L Z_H / D_{cm} = TKE, \quad (5)$$

Within a quasi-static approach one can not take into account the pre-scission kinetic energy. So, we include into TKE only the Coulomb repulsion energy. The TKE distribution accounts for the finite energy resolution through the parameter  $\Delta E$ .

$$Y(TKE) = \sum_{ijkl} P(\alpha_{1i}, \alpha_{3j}, \alpha_{4k}, \alpha_{6l}) \frac{e^{-\left(\frac{TKE_{ijkl} - TKE}{\Delta E}\right)^2}}{\sqrt{\pi} \Delta E}, \quad (6)$$

The first two moments are calculated with the following equations:

$$\langle TKE \rangle = \frac{\int TKE Y(TKE) dTKE}{\int Y(TKE) dTKE}, \quad (7)$$

and

$$\sigma_{TKE}^2 = \frac{\int (TKE - \langle TKE \rangle)^2 Y(TKE) dTKE}{\int Y(TKE) dTKE}, \quad (8)$$

## Predicted fission fragment mass and kinetic energy distributions

The model described in the previous section is now applied to fragment properties for superheavy nuclei for which spontaneous fission has been already detected and could therefore be re-measured with better statistics in the near future. These are  $^{264-278}$  Hs,  $^{268-280}$  Ds,  $^{276-286}$  Cn,  $^{285-287}$  Fl,  $^{290-293}$  Lv and  $^{294}$  Og [10-12].

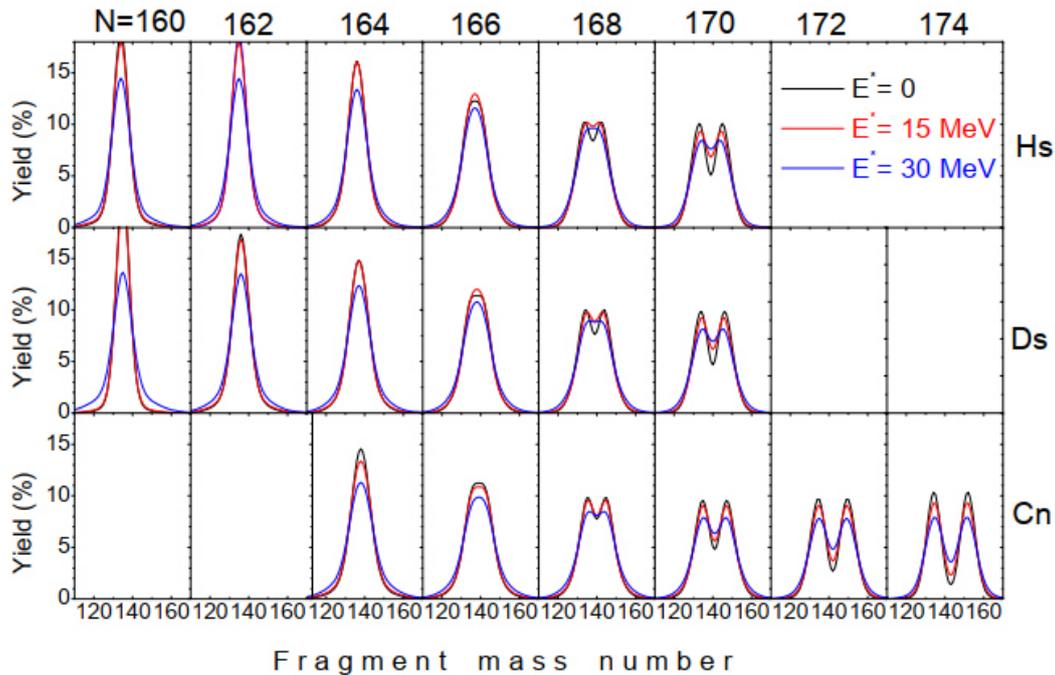


Figure 2. The calculated fragment mass distributions for isotopes of Hs, Ds and Cn for which spontaneous fission has been detected. Three values of the excitation energy  $E^*$  have been considered.  $T_{coll} = 2$  MeV,  $E_d = 40$  MeV.

The calculated mass distributions corresponding to Hs, Ds and Cn isotopes are presented in Figure 2. In all these three series, a transition from symmetric to asymmetric fission is predicted with increasing neutron number  $N$ . The transition point is  $N \approx 168$ . The "super-symmetric" fission occurs at  $N \approx 160$ . The dependence on the excitation energy (under assumption of constant entropy) is also shown.

There is no noticeable difference between  $E^* = 0$  and 15 MeV and very small between  $E^* = 15$  and 30 MeV. It is good news since the features discussed above are not expected to be washed out if these nuclei are produced with moderate excitation. So the mass distributions in the SHE region is quite robust with respect to the excitation energy of the fissioning nucleus. The case of the heaviest nuclei ever produced, Fl, Lv and Og, is presented in Figure 3. All these nuclei are predicted to fission into fragments with unequal masses. The dependence on the excitation energy is slightly stronger this time. As expected, the mass-symmetric yield increases with  $E^*$  but not enough to overturn the mass-asymmetric character of the distributions. Figure 4 shows the calculated mass distributions for a long series of Ds isotopes. The heaviest three isotopes have not been detected but they are candidates to be found together with Platinum in cosmic rays or in ores (as

eka-Pt) [13]. In agreement with the trends observed previously, they are predicted to fission asymmetrically.

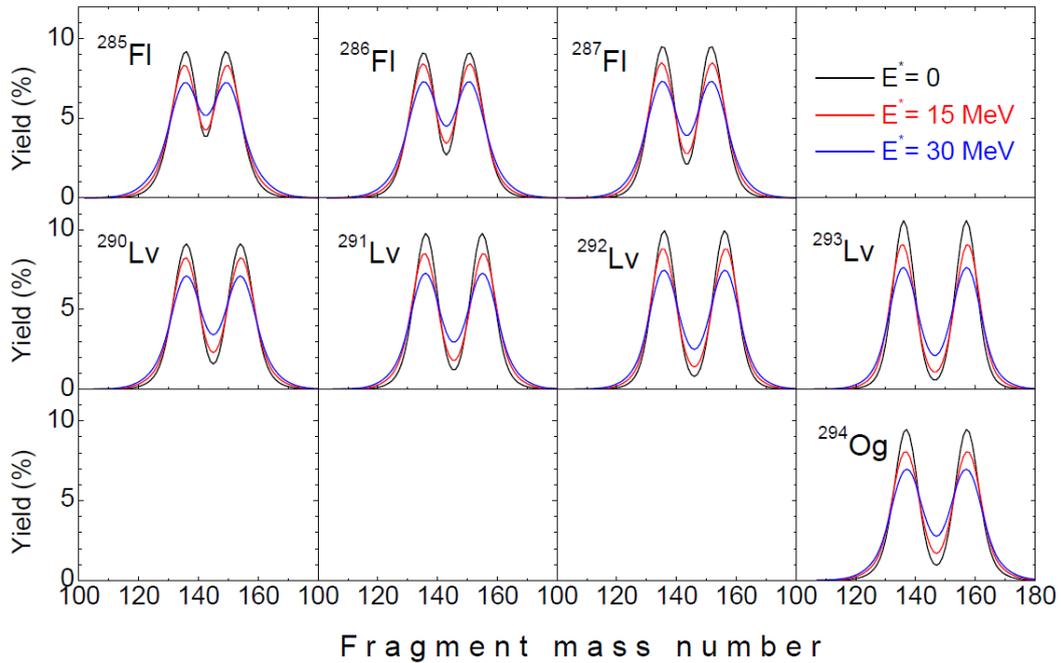


Figure 3. The same as in Figure 2 but for isotopes of Fl, Lv and Og.

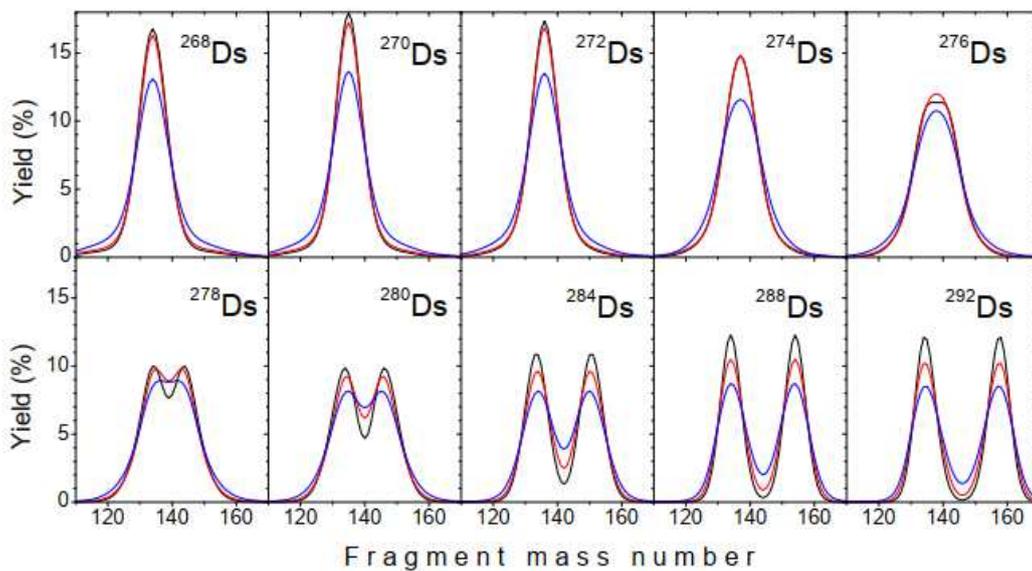


Figure 4. The same as in Figure 2 but for a larger series of Ds isotopes.

Let us now move to the total kinetic energy distributions. For all nuclei studied here their shapes are identical (quasi-gaussian) and therefore a similar presentation as for the mass distributions is not appropriate. Instead we show the first two moments of these distributions as a function of the mass  $A$  of the fissioning isotope in Figures 5 and 6, respectively.

The precission contribution to the average total kinetic energy is neglected meaning that the values given are lower limits. For each series of isotopes there is a slight decrease with  $A$  due to the increase of the radius ( $A^{1/3}$ ) and a decrease

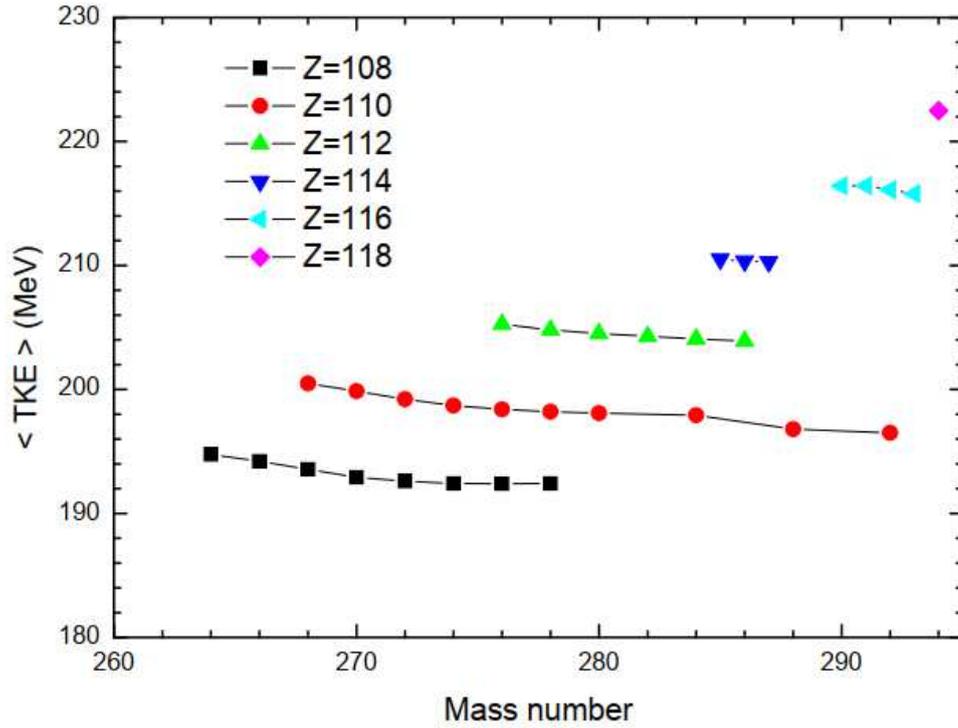


Figure 5. The average total kinetic energy of the fragments for the fission of isotopes of Hs, Ds, Cn, Fl, Lv and Og for which spontaneous fission has been detected.  $T_{coll} = 2$  MeV,  $\Delta E = 10$  MeV.

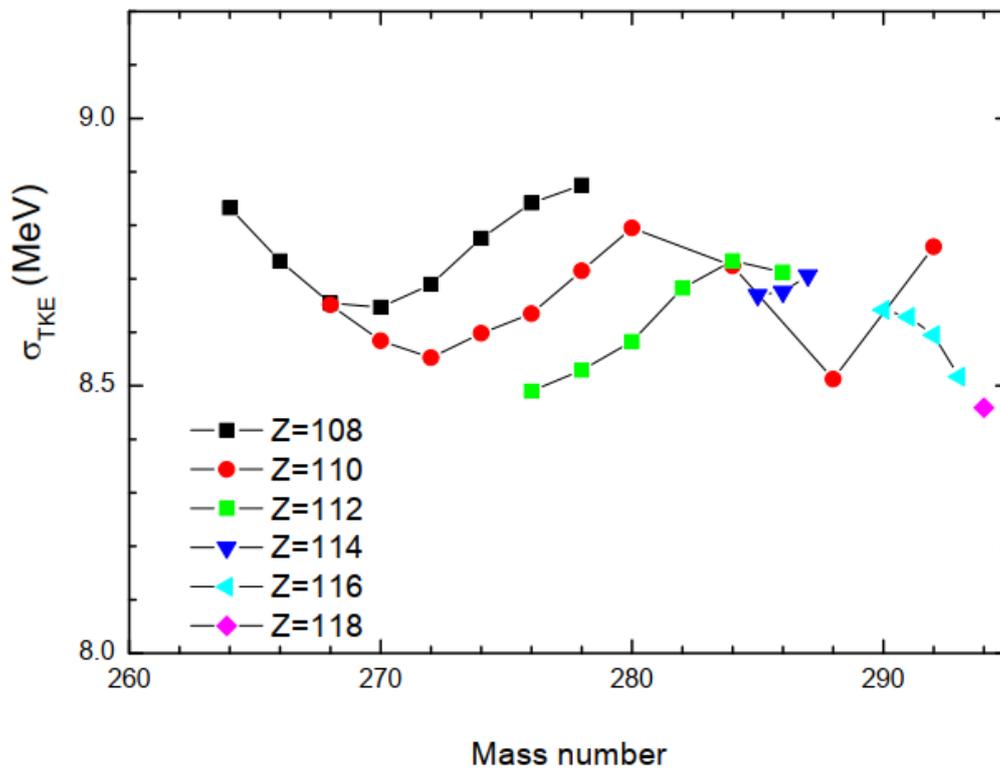


Figure 6. Second moments of the total kinetic energy distributions for the fission of isotopes of Hs, Ds, Cn, Fl, Lv and Og for which spontaneous fission has been detected.

of the product  $Z_L \times Z_H$ . Concerning the width of the TKE distributions, they exhibit a more complex variation with the mass of the isotope. For Hs and Ds

isotopes there is a minimum at  $N \approx 162$  close to the neutron number where the mass distributions are also the narrowest. It is a sign of a strong nuclear structure effect in extremely deformed nuclei (pre-scission shapes).

## Conclusion

The mass and TKE distributions of the fission fragments for the fission of selected even-even isotopes of Hs, Ds, Cn, Fl, Lv and Og are estimated using a pre-scission point model. The influence of the excitation energy of the fissioning system on these distributions is studied. The underlying potential energy surfaces are calculated with Strutinsky's shell correction procedure in a four dimensional deformation space using Cassini ovals as basic shapes.

With increasing neutron number  $N$ , a transition from symmetric to asymmetric fission is predicted at  $N \approx 168$ . Very narrow symmetric fission occurs at  $N \approx 160$ . It is a signature of a shell effect in extremely deformed nuclei. There is no dramatic change in the mass distributions when the excitation energy increases from 0 to 30 MeV; they are therefore quite robust. With increasing the total mass  $A$  of the fissioning nucleus, a slow decrease of the average total kinetic energy and a minimum (at  $N \approx 162$ ) of the width of the TKE distribution is found.

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