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Investigation of fragmentation reactions of exotic nuclei in a high-energy approximation

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Studies of the properties of nuclei remote from the "valley of nuclear stability" make it possible to predict the properties of new nuclides based on systematic accumulations of data on the structure of nuclear matter. New phenomena in the behavior of nuclear matter are also being discovered. Such cores are called "exotic". The production of exotic nuclei is a multi-stage process, during which various approaches of theoretical and experimental physics are applied. One of the ways to obtain nuclei are fragmentation reactions of relatively light nuclei with high energy (more than 100 MeV), as a result of which exotic nuclei with different A and Z can be obtained. In this article, a study of the applicability of the high-energy approximation (HEA) in modeling such direct nuclear reactions was conducted and the results of comparing this approach with the exact solution of the Schrodinger equation using the example of a rectangular potential barrier and a Gaussian potential barrier are presented. Comparison of different approaches provides an understanding of the limitations of their applicability for further study of the properties of nuclei in interaction with each other and for solving the Schrodinger equation with similar potentials considered.

Keywords: exotic nuclei; fragmentation reaction; high-energy approximation; potential barrier

Introduction

The study of the region of the isotope map remote from the so-called "valley of nuclear stability" is an urgent direction of modern research in nuclear physics, since the systematic accumulation of data on the structure of nuclear matter depending on A and Z allows us to refine theoretical models and make reliable predictions of the properties of new nuclides based on them. Secondly, new, often unusual phenomena are discovered in the behavior of nuclear matter: the effects of nuclear shells, new modes of nuclear decay, neutron halos, Borromian nuclei, soft excitation modes, etc., which are called exotic [1].

The area of exotic nuclei is very different from the "valley of stability". Moving away from this region to the boundaries of the isotope map is accompanied by a change in the ratio of nucleons in the nucleus (the number of protons or neutrons in the nucleus increases), which leads to a decrease in the binding energy of the nucleus. At some point, the value of the binding energy goes through zero and becomes negative. Thus, the core becomes nuclear unstable (unbound). On the isotope map, the regions of bound and unbound nuclei are separated by lines called "stability boundaries".

The process of studying the characteristics of nucleus-nuclear collisions of exotic, loosely coupled nuclei at the stability boundaries also requires a detailed study of the properties of nuclear matter, the structure and mechanisms of interaction in this area of the isotope map. The study of exotic nuclei is closely related to the development of high-level experimental facilities for producing beams of radioactive nuclei, and the development of new theoretical models for describing the nuclear structure [2-4]. On the task of comparing the results of calculations with experimental data, the theorist needs to work in the same team with the experimenter, since conducting experiments with exotic nuclei is a rather complex and multi-stage process.

In this paper, the applicability of the high-energy approximation to describe the fragmentation reactions of exotic nuclei was investigated. In fragmentation reactions of relatively light nuclei with high energy (more than 100 MeV), exotic nuclei with different A and Z can be obtained in experiments [5, 6]. With an increase in the energy value of the incoming particle beam to several GeV, the cross-sections of the formation of secondary nuclei also increase. In this article, the applicability of the high-energy approximation (HEA) [7] was investigated in solving problems of modeling direct nuclear reactions and the results of comparing this approach with the exact solution of the Schrodinger equation on the example of a rectangular potential barrier and a Gaussian potential barrier are presented.

The analytical calculation method allows us to consider the limitations that arise when using two methods on simple models of nuclear interaction. A well-known problem of quantum theory, with which it is convenient to start searching for a solution to the problem, is the problem of direct scattering on a one-dimensional potential barrier. Rectangular or stepped barriers with the property of a sharp change in the value of the potential at the boundaries act as such a barrier. To simulate more realistic scattering taking into account the short-range strong interaction, the Schrodinger equation with a Gaussian potential is often used, the value of which changes smoothly at the boundaries. Since the problem of passing a particle through a potential barrier has a solution of the Schrodinger equation for a rectangular barrier and is well studied [8], comparison of this solution with the solution in the framework of the HEA for rectangular and Gaussian potentials provides an understanding of the limitations of their applicability for further study of the properties of nuclei in interaction with each other and for solving the Schrodinger equation with similar considered potentials.

High-energy approximation

Research in the field of high energies, when the collision of two nuclei occurs as a rapid peripheral process, has been conducted for many years in order to obtain exotic nuclei in a wide range of A and Z. Studies of fragmentation reactions, which are characterized by a high level of isotope yield in comparison with other reactions, are especially important for these purposes, which is explained by the selected energy range of incoming particles and a fairly small angular distribution of reaction products relative to the initial beam direction [9].

Various theoretical approaches are used to study nuclear reactions at high energies, among which one can single out the so-called "high-energy approximation" (HEA), which, despite a number of restrictions imposed on it, makes it possible to more accurately estimate the intensity of the predominant part of the scattering [10].

In this paper, the one-dimensional scattering problem was studied in the framework of a high-energy approach and compared with the exact solution of the Schrodinger equation for the problem of passing a particle through potential barriers of simple types (rectangular barrier and Gaussian potential barrier). The one-dimensional scattering problem has special properties that must be taken into account. The scattering process can occur only in two directions: either to maintain the direction of movement of the particle forward, or to change the direction of movement when reflected backward. While this makes the problem a bit unrealistic, it has the advantage of being mathematically more transparent.

The one - dimensional Schrodinger equation has the form

$$(\frac{d^2}{dx^2}+k^2)\Psi(x)=\frac{2m}{\hbar}V(x)\Psi(x) \tag{1}$$

Now we assume that the energy of the incident particle significantly exceeds the magnitude of the potential $V(x)$, and is also large enough that the wavelength of the particle is much smaller than the width of the potential a

$$\frac{V}{E}\ll 1, \quad ka\gg 1 \tag{2}$$

(In order-of-magnitude ratios such as this, the symbol V should be interpreted as a measure of the absolute magnitude of the potential). Under these conditions, we can assume that the backscattering will be very weak, that the wave function of the particle can be written in the form of $\Psi(x)$ in a good approximation:

$$\Psi(x)=\exp{(ikx)}\varphi(x), \tag{3}$$

where $\varphi(x)$ is a function that slowly changes depending on the wavelength of the particle.

Substituting 3 into the Schrodinger equation 1, we get

$$\left(2ik\frac{d}{dx}+\frac{d^2}{dx^2}\right)\varphi(x)=\frac{2m}{\hbar^2}V(x)\varphi(x) \tag{4}$$

Now the approximation is to discard the $\frac{d^2}{dx^2}$ – term of the equation, which assumes that φ changes slowly depending on the wavelength. In this case, the equation reduces to

$$\frac{d\varphi}{dx}=-\frac{i}{\hbar v}V(x)\varphi(x) \tag{5}$$

Now, if the equation $\Psi(x)=\exp(ikx)\varphi(x)$ reduces to an incident plane wave at a point $x=-\infty$ (i.e. if backscattering is neglected), then $\varphi(-\infty)=1$ is required as a boundary condition. Thus, we get:

$$\varphi(x)=\exp\left(-\frac{i}{\hbar v}\int_{-\infty}^{x}V(x')dx'\right) \tag{6}$$

And the wave function takes the form:

$$\Psi(x)=\exp\left(ikx-\frac{i}{\hbar v}\int_{-\infty}^{x}V(x')dx'\right) \tag{7}$$

Results and discussion

Rectangular potential barrier

Let us compare the exact solution of the Schrodinger equation with a high-energy approximation in the case of a simple one-dimensional motion of a particle through a rectangular potential barrier of the form:

$$V(x)=\begin{cases}V_0 & \text{if } 0\leq x\leq a\\ 0 & \text{if } x<0 \text{ and } x>a\end{cases} \tag{8}$$

The high-energy approximation [10] implies the passage of a particle with mass m and energy E of the potential barrier $V(x)$ in the case when the energy value significantly exceeds the value of the potential $E\gg V$. In this case, for the exact solution of the Schrodinger equation, it is fair to consider the case when the energy of the particle exceeds the height of the potential barrier $E\gg V$, to compare the two solutions.

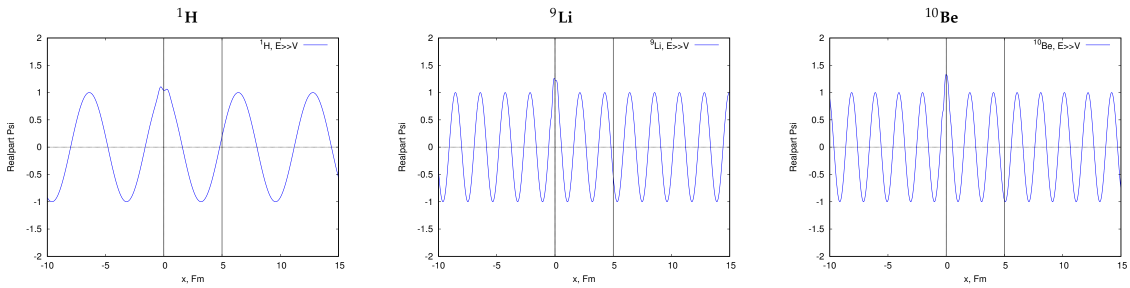
Solving in this case the stationary Schrodinger equation 1, we obtain as a result a system of equations:

$$\begin{aligned}&\psi_1=A_1\exp(ik_1x)+B_1\exp(-ik_1x), \quad if\ x<0\\&\psi_2=A_2\exp(ik_2x)+B_2\exp(-ik_2x), \quad if\ 0\leq x\leq a\\&\psi_3=A_3\exp(ik_1x), \quad if\ x>a\end{aligned} \tag{9}$$

where $k_1=\sqrt{\frac{2mE}{\hbar}}$, $k_2=\sqrt{\frac{2m(E-V(x))}{\hbar}}$, a is the width of the potential barrier.

According to the conditions of crosslinking of wave functions and their derivatives at the boundaries of the potential barrier at $x=0$ and $x=a$, we obtain the following coefficients and the corresponding type of wave functions:

$$\begin{aligned}
\psi_1 &= \exp\left(ik_1x\right) + \frac{\left(\left(k_1^2-k_2^2\right)\exp\left(-ik_2a\right) - \left(k_1^2-k_2^2\right)\exp\left(ik_2a\right)\right)}{\left((k_1+k_2)^2\exp(-ik_2a) - (k_1-k_2)^2\exp(ik_2a)\right)}\exp(-ik_1x),\\
\psi_2 &= \frac{(2k_1(k_1+k_2)\exp(-ik_2a))}{((k_1+k_2)^2\exp(-ik_2a)-(k_1-k_2)^2\exp(ik_2a))}\exp(ik_2x)+\\
&\quad \frac{(-2k_1(k_1-k_2)\exp(ik_2a))}{((k_1+k_2)^2\exp(-ik_2a)-(k_1-k_2)^2\exp(ik_2a))}\exp(-ik_2x),\\
\psi_3 &= \frac{(4k_1k_2\exp(-ik_1a))}{((k_1+k_2)^2\exp(-ik_2a)-(k_1-k_2)^2\exp(ik_2a))}\exp(ik_1x)
\end{aligned} \tag{10}$$



a) b) c)

Figure 1. Passage of a particle through a rectangular potential barrier if $E \gg V$: a) ^{1}H , b) ^{9}Li , c) ^{10}Be .

Figure 1 shows the solution of the Schrodinger equation when scattering on a rectangular barrier in the case when the energy of the incident particle exceeds the value of the potential $E \gg V$. The solutions were considered in comparison for different particles used in experiments to study fragmentation reactions as a beam hitting the target: using high-energy protons in Figure 1(a), in the reaction $^{9}\mathrm{Li} +^{9}\mathrm{Be} \rightarrow^{8}\mathrm{He} + p$ in Figure1(b), as well as in the reaction $^{10}\mathrm{Be} +^{9}\mathrm{Be} \rightarrow^{8}\mathrm{He} + 2p$ in Figure 1(c).

Let's consider a one-dimensional scattering problem in the case when the energy of particles hitting the barrier significantly exceeds the value of the potential, it is possible to use a high-energy approximation. Then the solution of the Schrodinger equation according to 7 will take the form:

$$\begin{aligned}
\psi_1 &= A_1\exp(ikx), \quad if\ x<0,\\
\psi_2 &= A_2\exp(ikx - \frac{i}{\hbar v}\int_{-\infty}^{x} V(x')dx'), \quad if\ 0 \le x \le a,\\
\psi_3 &= A_3\exp(ikx + \delta(x)), \quad if\ x>a
\end{aligned} \tag{11}$$

where $\varphi(x)$ is the scattering phase.

According to the law of conservation of the number of particles falling on a potential barrier:

$$R + D = 1, \quad R = \frac{|j_R|}{j_I}, \quad D = \frac{j_R}{j_I} \tag{12}$$

where R is the coefficient of reflection of the wave from the barrier, and D is the coefficient of passage of the wave through the barrier.

Knowing at the same time that the flux density vector in these formulas is expressed in terms of the wave function:

$$\vec{j} = \frac{i\hbar}{2m}\left[\psi\nabla\psi^* - \psi^*\nabla\psi\right] \tag{13}$$

Let's find the type of wave functions, when passing a potential barrier by a particle:

$$\begin{aligned}\psi_1 &= \exp(ikx),\\ \psi_2 &= \exp\left(ikx - \frac{iVmx}{\hbar^2 k}\right),\\ \psi_3 &= \exp\left(ikx - \frac{iVma}{\hbar^2 k}\right)\end{aligned} \tag{14}$$

where $k = \frac{\sqrt{2mE}}{\hbar}$, and the expression $-\frac{iVma}{\hbar^2 k}$ is the phase $\varphi(x)$, which persists after passing the barrier at distances $x \to \infty$.

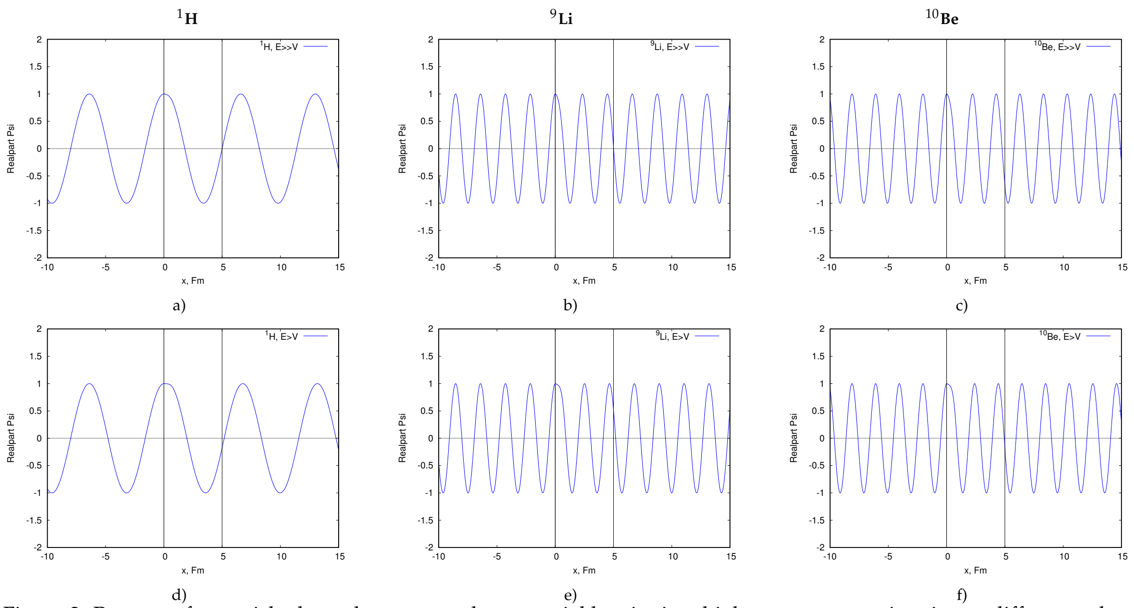


Figure 2. Passage of a particle through a rectangular potential barrier in a high-energy approximation at different values of the barrier height: 1. ^{1}H : a) $E \gg V$, d) $E > V$; 2. ^{9}**Li** : b) $E \gg V$, e) $E > V$; 3. ^{10}**Be** : c) $E \gg V$, f) $E > V$.

Figure 2 shows the solution of the Schrodinger equation for scattering on a rectangular barrier in the case of a high-energy approximation: 1) when the energy of the incoming particle exceeds the value of the potential $E \gg V$, 2) when the energy of the incoming particle is close to the value of the potential $E > V$. In comparison, one can see the best fulfillment of the condition of "stitching" wave functions at the boundaries of the potential barrier in the case when $E \gg V$.

Gaussian potential barrier

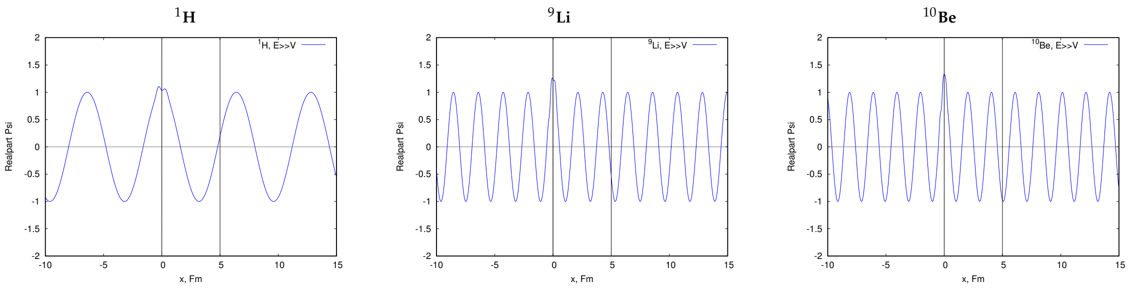
The one-dimensional Gaussian potential:

$$V(x) = V_0 \exp\left(-ax^2\right) \tag{15}$$

as well as the rectangular potential, is often chosen to illustrate simple quantum mechanical phenomena. Within the framework of the analytical solution, the form of the system of equations (10), for the Schrodinger equation with a rectangular potential is similar to the solution with a Gaussian potential:

$$
\begin{aligned}
\psi_1 = & \exp(ik_1x)+ \\
& \frac{\left(k_1^2-k_2^2\right)\exp(-ik_2a)-\left(k_1^2-k_2^2\right)\exp(ik_2a)}{(k_1+k_2)^2\exp(-ik_2a)-(k_1-k_2)^2\exp(ik_2a)}\exp(-ik_1x), \\
\psi_2 = & \frac{2k_1\,(k_1+k_2)\exp(-ik_2a)}{(k_1+k_2)^2\exp(-ik_2a)-(k_1-k_2)^2\exp(ik_2a)}\exp(ik_2x)+ \\
& \frac{-2k_1\,(k_1-k_2)\exp(ik_2a)}{(k_1+k_2)^2\exp(-ik_2a)-(k_1-k_2)^2\exp(ik_2a)}\exp(-ik_2x), \\
\psi_3 = & \frac{4k_1k_2\exp(-ik_1a)}{(k_1+k_2)^2\exp(-ik_2a)-(k_1-k_2)^2\exp(ik_2a)}\exp(ik_1x)
\end{aligned}
\tag{16}
$$

where $k_1 = \frac{\sqrt{2mE}}{\hbar}$, and k_2 takes the form $k_2 = \frac{\sqrt{2m(E-V_0\exp(-ax^2))}}{\hbar}$, and a is the width of the potential barrier.



a) b) c)

Figure 3. Passage of a particle through a gaussian potential barrier if $E \gg V$: a) ^{1}H , b) ^{9}Li , c) ^{10}Be .

Within the framework of the high-energy approximation, the solution of the Schrodinger equation with a Gaussian potential, taking into account the normalization for the flux density, takes the form:

$$
\begin{aligned}
\psi_1 &= \exp(ikx), \\
\psi_2 &= \exp\left(ikx - \frac{iV_0m}{2\hbar^2k}\sqrt{\frac{\pi}{a}}\left(\mathrm{erf}\,(\sqrt{a}x)+1\right)\right), \\
\psi_3 &= \exp\left(ikx - \frac{iV_0m}{2\hbar^2k}\sqrt{\frac{\pi}{a}}\left(\mathrm{erf}\,(a^{3/2})+1\right)\right)
\end{aligned}
\tag{17}
$$

where $\varphi(x) = -\frac{iV_0m}{2\hbar^2k}\sqrt{\frac{\pi}{a}}(\mathrm{erf}\,(a^{3/2})+1)$ is t he phase of the wave function. A comparison of Figures 3 and 4 shows that the high-energy approximation works much better when choosing a smoother Gaussian potential, since, unlike the rectangular potential barrier, this model is more close to the real model of the core. Figures 1-4 demonstrate the applicability of two approaches to solving the problem of passing a potential barrier by a particle at energy values of an incoming particle significantly exceeding the value of the potential. Figure 5

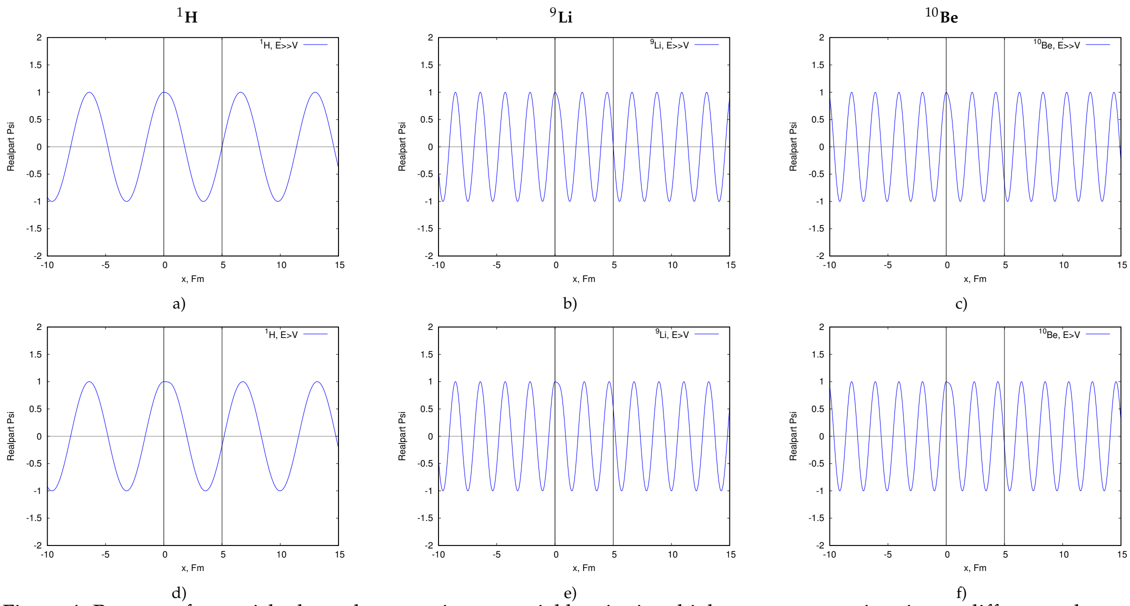


Figure 4. Passage of a particle through a gaussian potential barrier in a high-energy approximation at different values of the barrier height: 1. ^{1}H : a) $E \gg V$, d) $E > V$; 2. ^{9}**Li** : b) $E \gg V$, e) $E > V$; 3. ^{10}**Be** : c) $E \gg V$, f) $E > V$.

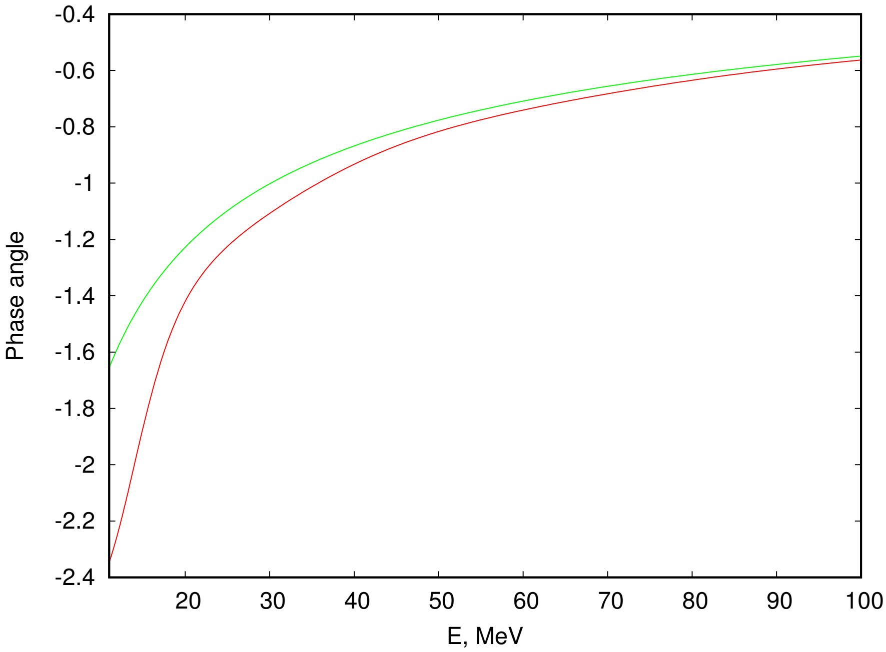


Figure 5. The dependence of the phase angle on the energy of particles impinging on a rectangular potential barrier: green shows the phase with an exact solution, red - with a high-energy approximation

shows the dependence of the phase of the wave function that occurs during the passage of the barrier: Figure 5 shows that at high energies, the solution in the high-energy approximation is close to the exact solution, despite the limitations imposed due to the neglect of backscattering from the walls of the potential barrier.

Conclusions

In this paper, the applicability of the high-energy approximation in solving problems of modeling direct nuclear reactions was investigated, in particular in the framework of studying the fragmentation reactions of exotic nuclei, and the results of comparing this approach with the exact solution of the Schrodinger equation on the example of simple types of potential barrier are presented. To solve the problem of direct scattering on a one-dimensional potential barrier, the interaction of particles with a rectangular barrier with the property of a sharp change in the value of the potential at the boundaries was considered, and to simulate a more realistic scattering taking into account the short-range strong interaction, the interaction with a Gaussian potential was considered, the change in the value of which at the boundaries occurs smoothly. Comparison of the two solutions provides an understanding of the limitations of their applicability for further study of the properties of nuclei in interaction with each other and for solving the Schrodinger equation with similar potentials considered.

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