

Temperature Dependent Hadronic Bag and QGP Phase Transition in Dual QCD

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Based on the magnetic symmetry structure of non-Abelian gauge theories, a dual QCD gauge theory has been constructed which takes into account the local structure as well as the topological features of the color gauge group into its dynamics in a completely dual-symmetric way. Using such dual version of QCD in thermal domain following the partition function approach and the grand canonical ensemble formulation, the phase transition from hadron to QGP phase has been investigated within the framework of temperature dependent hadronic bag in the entire $T - \mu$ plane. The various thermodynamic properties like pressure, energy density, speed of sound and specific heat of the hadron/QGP phase have been evaluated and shown to lead an evidence for the first order phase transition. In the region around $T_c < T < 4T_c$, the specific heat and speed of sound are strongly influenced by the magnetically charged particles directly related to thermal monopoles evaporating from the magnetic condensate present at low temperature.

Keywords: dual QCD, thermal bag, phase transition, QGP.

Introduction

Soon after the discovery of Quantum Chromodynamics (QCD) [1, 2, 3] and following the realization that QCD exhibits asymptotic freedom [4] it was recognized that normal hadronic matter undergoes a phase transition, where the individual hadrons dissolve into their constituents and produce a collective form of matter known as the Quark-Gluon Plasma (QGP) [5] under extreme conditions of high temperature and low chemical potential (μ_B). The precise determination of the phase boundary between QGP and hadron gas (HG) at high T and bario-chemical potential (μ_B) has been a subject of intense research in recent years from experimental as well as theoretical point of view. In this connection, during past few years, the possibility of creating such high temperature QGP and studying QGP phase of matter by colliding heavy ions in the laboratory has been the main goal of experiments at Nuclotron-based ion collider facility (NICA) [6, 7] at Dubna, the Relativistic Heavy-Ion Collider (RHIC) [8, 9] at BNL and the Large Hadron Collider (LHC) [10, 11, 12] at CERN and needs some reliable theoretical explanation of various signals which depend on the pressure, entropy, transition temperature and the equation of state. Unfortunately, the theoretical description of the QGP from first principle is extremely difficult as the interaction between quarks and gluons as described by QCD, is strong, therefore, perturbative QCD is not applicable. Being in the non-perturbative domain, the lattice QCD [13, 14,

15, 16, 17] and the effective models remain the only tools to study the phase transition. The lattice-QCD simulation is, however, of limited practical use at finite baryon density due to the well known fermion sign problem. Therefore, the critical behavior of such matter with finite baryon chemical potential needs to be studied in the framework of QCD motivated phenomenological models such as MIT bag model [18] which provides a semi phenomenological description of an EOS that features a quark-hadron transition.

The present paper mainly deals with the investigation of thermodynamic properties of QGP using an infrared effective dual QCD formulation based on magnetic symmetry. In section "Dual QCD formalism with magnetic symmetry", the dual QCD formulation has been presented and analyzed to explain the confining behavior of QCD vacuum in its non-perturbative sector. In section "QGP-phase transition dynamics and thermal hadronic bags", the QGP-phase transformation dynamics with a thermal version of hadronic bags has been studied in order to investigate the thermodynamics quantities like the pressure, the energy density, the specific heat and the square of speed of sound in QGP phase. Calculation of critical parameters involves the use of Gibbs's criteria of thermodynamic equilibrium. The results and their implications have been discussed in section "Evaluation of thermodynamic and transport properties of QGP".

Dual QCD formalism with magnetic symmetry

Based on the magnetic symmetry structure of non-Abelian gauge groups, a dual gauge formulation [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. which produce the magnetic condensation dynamically for its vacuum and provides a gauge invariant investigation and topological ground of confinement, the resulting dual QCD has been a subject of utmost importance to analyze the Quark-gluon plasma phase of the hadronic matter. It is based on introducing the magnetic symmetry as an internal isometry H admitting some additional Killing vector fields which are internal such that H is a Cartan's subgroup of a gauge group G and commutes with it [20, 29]. It, in turn, restricts the associated gauge potential which may be expressed in the following form,

$$D_\mu \hat{m} = 0, i.e. (\delta_\mu + g \mathbf{W}_\mu \times) \hat{m} = 0, \quad (1)$$

where \mathbf{W}_μ is the gauge potential of the gauge group G . It leads to the following form given as $\mathbf{W}_\mu = A_\mu \hat{m} - g^{-1} (\hat{m} \times \delta_\mu \hat{m})$, where, $A_\mu \equiv \hat{m} \cdot \mathbf{W}_\mu$ is the Abelian component and \hat{m} may then be viewed to define the homotopy class of the mapping $\Pi_2(S^2)$ as, $\hat{m} : S_R^2 \rightarrow S^2 = SU(2)/U(1)$, where, S_R^2 is the two dimensional sphere of the three dimensional space and S^2 is the group coset space completely fixed by \hat{m} . Using the dual magnetic potential $B_\mu^{(d)}$ for the magnetic part in order to remove the problem of singular behavior of the potential associated with monopoles, the dual QCD Lagrangian is constructed in the following form [29].

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} B_{\mu\nu}^2 - \frac{1}{2} F_{\mu\nu} B^{\mu\nu} + \bar{\psi}_r i \gamma^\mu \left[\delta_\mu + \frac{g}{2i} (A_\mu + B_\mu^{(d)}) \right] \psi_r + \\ & + \bar{\psi}_b i \gamma^\mu \left[\delta_\mu - \frac{g}{2i} (A_\mu + B_\mu^{(d)}) \right] \psi_b + m_0 (\bar{\psi}_r \psi_r + \bar{\psi}_b \psi_b) + \end{aligned}$$

$$\left[\delta_\mu + i \frac{4\pi}{g} (A_\mu^{(d)} + B_\mu) \right] \varphi^2 - V(\varphi), \quad (2)$$

where $V(\varphi)$ is the effective potential responsible for the dynamical breaking of magnetic symmetry and has its quadratic form reliable in the phase transition study of dual QCD vacuum as given by,

$$V_{pt}(\varphi) = 3\lambda\alpha_s^{-2}(\varphi^* \varphi - \varphi_0^2)^2, \quad (3)$$

Using the cylindrical symmetry the field equations associated with the Lagrangian (2) in the quenched approximation may be derived in the following form [29],

$$\frac{1}{\rho} \frac{d}{d\rho} \left(\rho \frac{d\chi}{d\rho} \right) - \left[\left(\frac{n}{\rho} + (4\pi\alpha_s^{-1})^{1/2} B(\rho) \right)^2 + 6\lambda\alpha_s^{-2} (\chi^2 - \varphi_0^2) \right] \chi(\rho) = 0, \quad (4)$$

Table 1.

The masses of vector and scalar glueballs as a functions of α_s .

α_s	γ	$\phi_0(\text{GeV})$	$m_B(\text{GeV})$	$m_\phi(\text{GeV})$	$\lambda_{\text{QCD}}^{(d)}(\text{fm})$	$\xi_{\text{QCD}}^{(d)}(\text{fm})$	$k_{\text{QCD}}^{(d)}$
0.12	8.30	0.143	2.11	4.20	0.09	0.05	1.8
0.22	7.89	0.149	1.51	2.22	0.13	0.09	1.4
0.47	6.28	0.167	1.21	1.22	0.16	0.16	1
0.96	5.40	0.181	0.929	0.655	0.215	0.31	0.7

$$\frac{d}{d\rho} \left[\rho^{-1} \frac{d}{d\rho} (\rho B(\rho)) \right] - (16\pi\alpha_s^{-1})^{1/2} \left(\frac{n}{\rho} + (4\pi\alpha_s^{-1})^{1/2} B(\rho) \right) \chi^2(\rho) = 0, \quad (5)$$

Imposing the asymptotic boundary conditions appropriate for the large-scale behavior of QCD $B(\rho) \xrightarrow{\rho \rightarrow \infty} -\frac{ng}{4\pi\rho}$ and $\varphi \xrightarrow{\rho \rightarrow \infty} \varphi_0$ leads to the asymptotic solution for $B(\rho)$ as $B(\rho) = -\frac{ng}{4\pi\rho} [1 + F(\rho)]$, where the function $F(\rho)$, in asymptotic limit may then be obtained in the following form,

$$F(\rho) \xrightarrow{\rho \rightarrow \infty} C\sqrt{\rho} \exp(-m_B\rho), \quad (6)$$

Utilizing the asymptotic solutions of the associated dual QCD fields [29, 30], the energy per unit length of the resulting flux tube configuration may be given as,

$$k = 2\pi \int_0^\infty \rho d\rho \left\{ \frac{n^2 g^2}{32\pi^2 \rho^2} \left(\frac{dF}{d\rho} \right)^2 + \frac{n^2}{\rho^2} F^2(\rho) \chi^2(\rho) + \left(\frac{d\chi}{d\rho} \right)^2 + \frac{48\pi^2}{g^4} \lambda (\varphi^* \varphi - \varphi_0^2)^2 \right\} = \gamma \varphi_0^2, \quad (7)$$

The associated vector and scalar glueball masses generated after the dynamical breaking of magnetic symmetry may be evaluated using relations, $\frac{m_\phi}{m_B} = \sqrt{3}(2\pi\alpha_s)^{-1/2}$ and $\alpha' = \frac{1}{2\pi k} = \frac{1}{2\pi\gamma\varphi_0^2}$ associated with the string tension and

the Regge slope parameter. The numerical results of the glueball masses, characteristic length scales and Ginzburg-Landau (GL) parameter [20] are shown in table 1. The estimate for the GL parameter ($\kappa_{QCD}^{(d)}$) demonstrate different superconducting behavior (type-I, II) of the QCD vacuum intimately connected to the process of quark confinement in the low energy sector. It is therefore, strongly desired to construct a low energy effective theory of QCD which might explain the correct confining features of QCD on one hand and complete phase transition behavior on the other hand, in an effective way.

QGP-phase transition dynamics and thermal hadronic bags

Phase transitions are associated with the evolution of thermodynamic quantities such as pressure, energy density and entropy density, as well as a set of response functions, like, specific heat and speed of sound. In order to study the phase transition dynamics, let us begin with a relativistic quantum system where particles can be efficiently created and destroyed. Using the grand canonical ensemble formalism the corresponding partition function is given in the following form,

$$Z = Tr \left[\exp \left(-\frac{1}{T} (\hat{H} - \mu \hat{N}) \right) \right], \quad (8)$$

where, $\beta = 1/T$ and \hat{H} and \hat{N} is the Hamiltonian and the particle number operator respectively. The pressure P and the energy density ϵ for a spatial homogeneous system are given by the first derivatives of $\ln Z$ as,

$$P = T \left(\frac{\delta \ln Z}{\delta V} \right)_T, \epsilon = \left(\frac{T^2}{V} \right) \left(\frac{\delta \ln Z}{\delta T} \right)_V, \quad (9)$$

Furthermore, other observables of phenomenological interest derivable from the basic thermodynamic quantities P and ϵ are the normalized specific heat and square of speed of sound expressed in the following form,

$$\frac{C_V}{T^3} = 4 \frac{\epsilon(T)}{T^4} + T \left[\frac{\delta(\epsilon/T^4)}{\delta T} \right]_V, c_s^2 = \frac{dP}{d\epsilon} = \epsilon \frac{d(P/\epsilon)}{d\epsilon} + \frac{P}{\epsilon}, \quad (10)$$

In order to calculate the various thermodynamic quantities let us start with the resulting expression for bosonic partition function written as [32],

$$\ln Z_b(T, \mu) = \frac{g_b V}{2\pi^2} \int_0^\infty dk k^2 \ln \left[1 - \exp(-\beta \sqrt{k^2 + m_b^2}) \right], \quad (11)$$

while that for fermions is [32],

$$\begin{aligned} \ln Z_f(T, \mu) = & \frac{g_f V}{2\pi^2} \int_0^\infty dk k^2 \{ \ln [1 + \exp(-\beta(\sqrt{k^2 + m_f^2} - \mu))] + \\ & + \ln [1 + \exp(-\beta(\sqrt{k^2 + m_f^2} + \mu))] \}, \end{aligned} \quad (12)$$

g_b and g_f are the degeneracy factors for bosons and fermions, respectively, and m_b and m_f denote the masses of the particles. Hence, using the above expression

(12) with the degeneracy factor $g_f = 2 \times 2$ for nucleons, the pressure and energy density for the hadronic matter is expressed in the following form,

$$P_h = \frac{7}{180}\pi^2 T^4 + \frac{1}{6}\mu_q^2 T^2 + \frac{1}{12\pi^2}\mu_q^4, \quad \epsilon_h = \frac{7}{60}\pi^2 T^4 + \frac{1}{2}\mu_q^2 T^2 + \frac{1}{4\pi^2}\mu_q^4, \quad (13)$$

Having a description of the hadronic phase at hand, it is straightforward to analyze the thermodynamic properties of the quark-gluon plasma and to study the dynamics of hadron to quark-gluon phase transition. Possibly, the simplest model to describe an approximate physics describing the matter where quark and gluons are the proper degrees of freedom of the system is the MIT bag model [18]. The non-trivial feature of the bag model EOS is that the vacuum pressure generates positive and large contribution to the bag energy density as given by,

$$E_h = BV + \frac{C}{R_h}, \quad (14)$$

where, B represents the bag constant which gives the bag pressure. Hence, the grand canonical partition function for the QGP phase consisting of a perturbatively interacting gas of quarks and gluons can be written in the following form,

$$\ln Z_{QGP} = \ln Z_{QGP}^0 + \ln Z_{QGP}^{Vac}, \quad (15)$$

where, in the r.h.s the first term is the contribution at free level while the second term represents the non-perturbative vacuum contribution in the form of a T-dependent bag constant B(T) which may be identified by the confining part of the energy expression recalculated after taking the multi-flux tube system as a periodic system on a S^2 -sphere [30, 32] and is given by,

$$B^{1/4}(T) = \left(\frac{12}{\pi^2}\right)^{1/4} \frac{m_B^{(T)}}{8}, \quad (16)$$

where $m_B^{(T)}$ B is the thermal vector glueball mass derived using path-integral formalism alongwith the mean-field approach [31] and is given by the following expression,

$$m_B^{(T)} = \left[m_B^{(0)2} - (8\pi^2 + 2\pi\alpha_s) \frac{T^2}{3} \right]^{1/2}, \quad (17)$$

where $m_B^{(0)}$ is the non-thermal contribution to the vector glueball mass. With these considerations, resulting expressions for the energy density and pressure for QGP phase is then given as,

$$\epsilon_p = \frac{2}{3}\pi^2 T^4 + 2\mu_q^2 T^2 + \frac{\mu_q^4}{\pi^2} + B(T), \quad P_p = \frac{2}{9}\pi^2 T^4 + \frac{2}{3}T^2 \mu_q^2 + \frac{\mu_q^4}{3\pi^2} - B(T), \quad (18)$$

The thermodynamical phase stability then requires the equality of pressure, chemical potential at the boundary of two phases i.e., $P_h = P_p = P_c$, $\mu = 3\mu_q = \mu_c$ at the transition point $T_p = T_h = T_c$. Since at the critical temperature the additional degrees of freedom carried by the quark-gluon plasma, are to be released which results in an increase in the thermodynamic quantities and is investigated in the next section.

Evaluation of thermodynamic and transport properties of QGP

The critical parameters of QGP phase transition within the framework of thermal bag may be evaluated using above mentioned phase equilibrium criteria due to Gibbs which may be used with the 3-d pressure as depicted in Figure 1(a) for different coupling values in infrared sector of QCD. The change of the quark-chemical potential from zero to non-zero values defines uniquely and precisely the QGP phase transition temperatures and their values are found to decrease with increasing values of chemical potential. At low values of μ and T the nuclear matter is composed of confined hadrons, but with increasing T or μ , the hadronic matter undergoes a phase transition towards a plasma of deconfined quarks and gluons. At vanishing chemical potential ($\mu_q = 0$) the transition temperature of 0.187 GeV, 0.140 GeV, 0.116 GeV and 0.090 GeV for the case of couplings $\alpha_s = 0.12; 0.22; 0.47$ and 0.96 are obtained respectively.

In Figure 1(b), the variation of energy density for hadron and QGP phase has been depicted and it has been observed that the energy density increases abruptly at the phase transition temperature followed by a finite jump discontinuity indicating the rapid rise in the degree of freedom carried by quarks and gluons. The size of the discontinuity in the energy density, the so-called latent heat ($\Delta\epsilon$) is given by,

$$\Delta\epsilon = \epsilon_p(T_c) - \epsilon_h(T_c) = \frac{33\pi^2 T_c^4}{60} + \frac{3}{2}\mu_q^2 T_c^2 + \frac{3}{4\pi^2}\mu_q^4 + B(T), \quad (19)$$

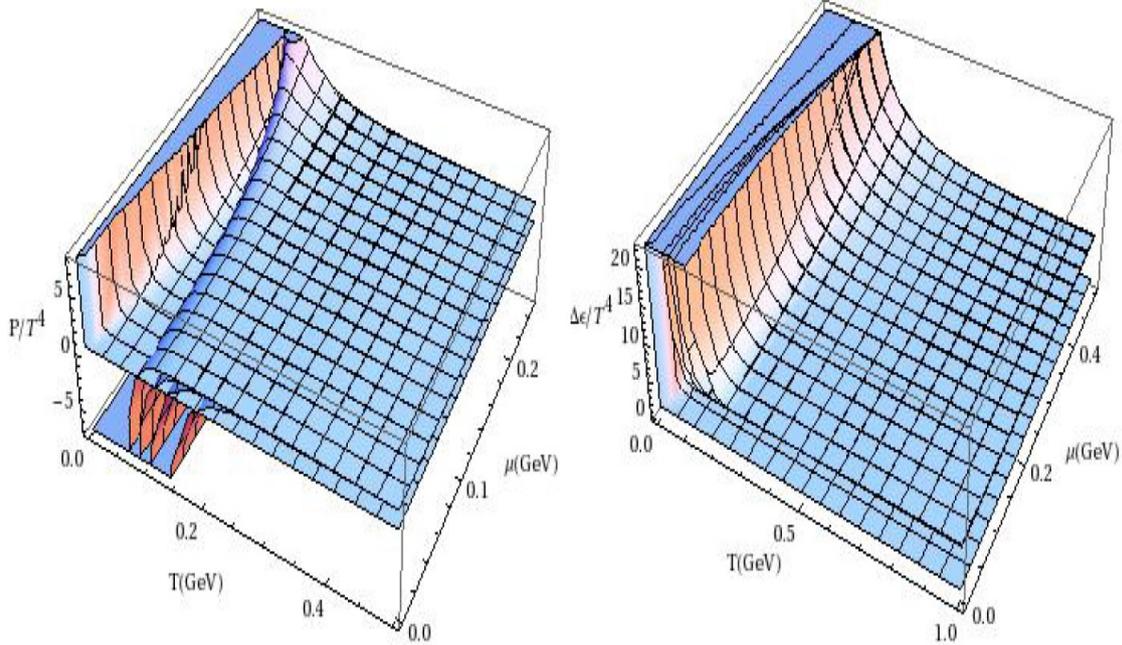


Figure 1. The variation of normalized pressure and energy density for plasma and hadron phase in $T - \mu$ plane using thermal bag for $\alpha_s = 0.12; 0.22; 0.47$ and 0.96 coupling, respectively.

The value of $\Delta\epsilon/T_c^4$ at the transition temperature (T_c) is found to increase with the increase in chemical potential due to the decrease in the transition temperature T_c . For vanishing chemical potential the numerical value of $\Delta\epsilon$ comes out to be $1.17 \text{ GeV}/\text{fm}^3$, $0.38 \text{ GeV}/\text{fm}^3$, $0.17 \text{ GeV}/\text{fm}^3$ and $0.06 \text{ GeV}/\text{fm}^3$ for different coupling $\alpha_s = 0.12, 0.22, 0.47$ and 0.96 respectively.

Another important thermodynamic quantity characterizing the equation of state for a system undergoing phase transition is the specific heat which is a measure of energy fluctuations in the system and these fluctuations tend to rise sharply near a phase transition as shown in Figure 2. The thermal evolution of normalized specific heat shows a small upward cusp at T_c and above the critical temperature specific heat rises with the temperature due to emergent chromomagnetic monopoles. In this scenario, magnetically charged particles are important component above the phase transition, possibly contributing to the physical properties of the strongly interacting QGP [34, 35, 36, 37, 38]. These component of the deconfined plasma are, in turn, directly related to thermal monopoles evaporating from the magnetic condensate present at low temperature. Furthermore, another observable of phenomenological interest is the square of speed of sound in the hot medium related to the speed of small perturbations produced in the QCD matter. The thermal evolution of the square of speed of sound with temperature and chemical potential has been depicted in Figure 3 and a first order phase transition is indicated as a result of a sudden drop of c_s^2 around transition temperature alongwith a further rise with temperature approaching to the value $c_s^2 = 0.33$ near T_c . In the region around $T_c < T < 4T_c$, c_s^2 decreases with temperature and in this scenario, the near T_c QCD matter is considered as a semi-quark-gluon-monopole plasma (sQGMP) [39] that contains not only electrically charged quasi-particles, quark and gluons, but also magnetically charged quasi particles, monopoles. In conclusion, this scenario emphasizes the change in chromodegrees of freedom, and recasts the QCD phase diagram into electrically and magnetically dominated regimes.

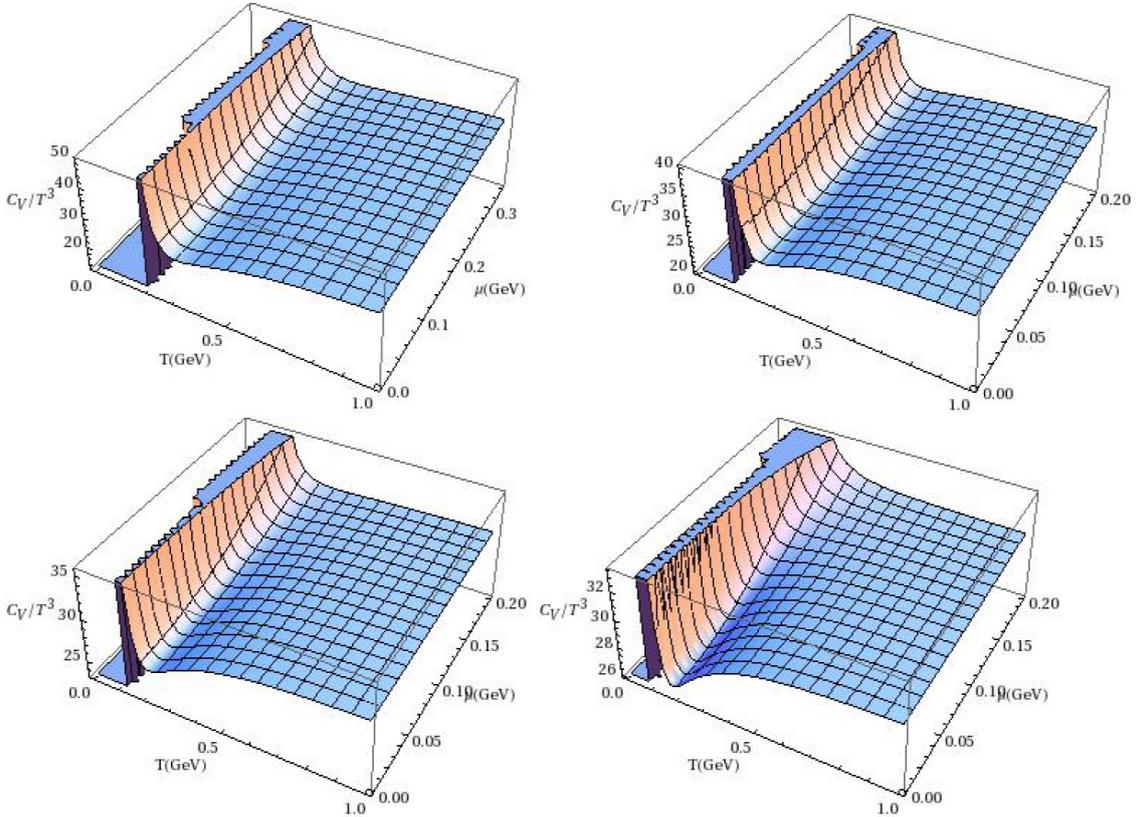


Figure 2. The variation of normalized specific heat for QGP phase in $T - \mu$ plane using thermal bag for $\alpha_s = 0.12, 0.22, 0.47$ and 0.96 coupling respectively.

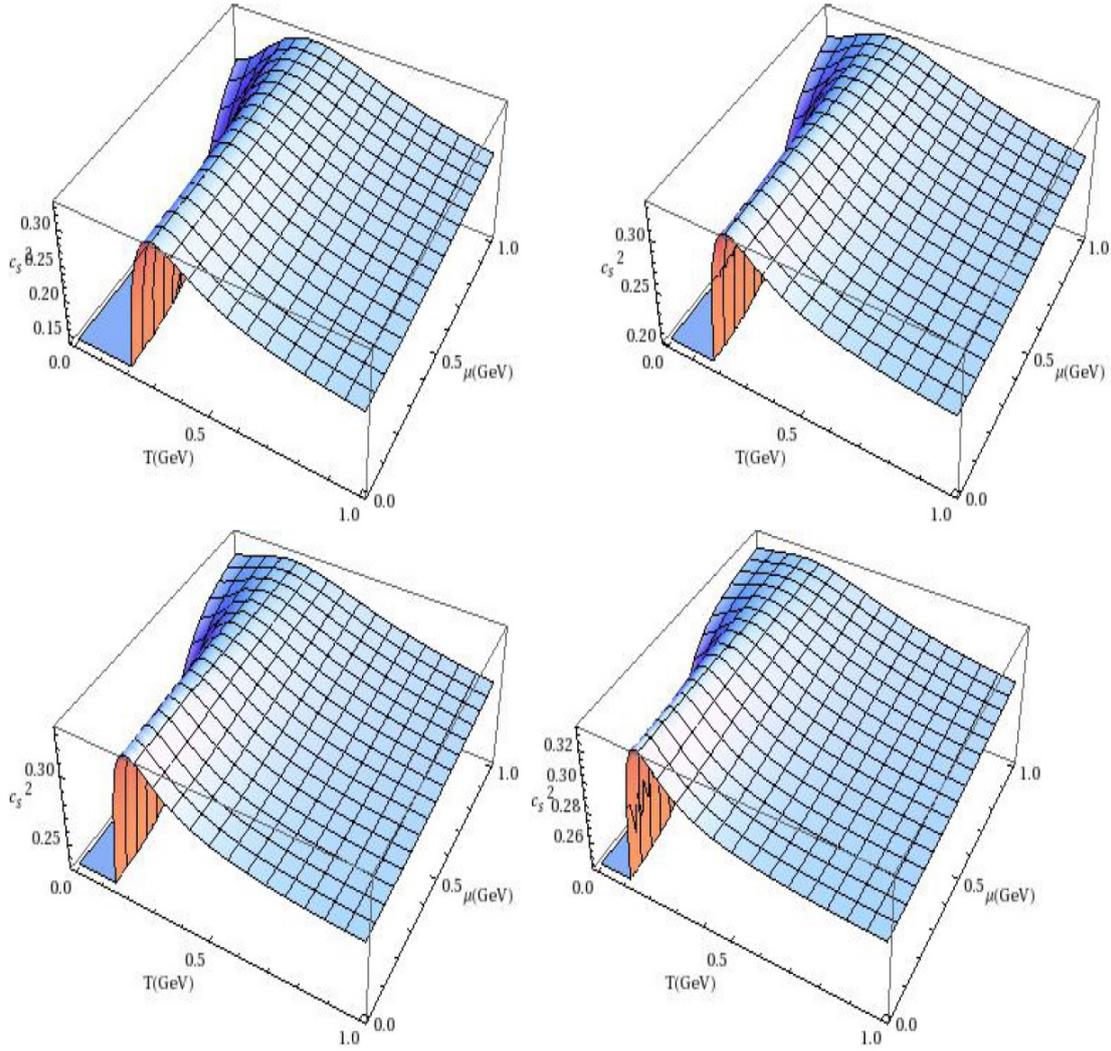


Figure 3. The variation of speed of sound for QGP phase in $T - \mu$ plane using thermal bag for $\alpha_s = 0.12, 0.22, 0.47$ and 0.96 coupling respectively.

Conclusion

Based on the magnetic symmetry structure of non-Abelian gauge theories, a dual QCD gauge theory has been constructed which takes into account the local structure as well as the topological features of the color gauge group into its dynamics in a completely dual-symmetric way. Using such dual version of QCD in thermal domain following the partition function approach and the grand canonical ensemble formulation, the phase transition from hadron to QGP phase has been investigated within the framework of temperature dependent hadronic bag in the entire $T - \mu$ plane. The various thermodynamic properties like pressure, energy density, speed of sound and specific heat of the hadron/QGP phase have been evaluated and shown to lead an evidence for the first order phase transition. In the region around $T_c < T < 4T_c$, the specific heat and speed of sound are strongly influenced by the magnetically charged particles directly related to thermal monopoles evaporating from the magnetic condensate present at low temperature.

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