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Quark-hadron phase transition study in hadron-nucleus interactions in self-affine scaling scenario

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In the present work the phase transition and its dependence on target excitation has been studied in two dimensional ($\eta - \phi$) self affine space using the experimental data of pions obtained from π^- -AgBr interactions at 350 GeV/c. For studying target excitation dependence the data for produced pions are divided into three sets depending on the number of grey particles (n_g). The different sets corresponds to the different degrees of target excitation. The Levy indices μ measured from the analysis fulfills the requirement of the levy stable region $0 \leq \mu \leq 2$. The Levy index μ <1 indicates that a thermal phase transition may exist in the π^- -AgBr interactions at 350 GeV/c. Further the analysis indicates different degrees of multifractality for different target excitation. Moreover, the value of universal scaling exponent (ν) obtained from Ginzburg-Landau (GL) theory indicates that no evidence of second order phase transition has been found in the interaction.

Keywords: hadron-nucleus interaction; phase transition; target excitation; Levy index; Ginzburg – Landau theory; self-affine scaling.

Introduction

Quantum chromodynamics predicts that in high energy interactions, a new matter state–quark-gluon plasma (QGP)– may be formed. The newly produced hot system subsequently cools and undergoes a phase transition from the deconfined QGP to confined hadrons [1, 2]. The hadrons produced in such processes are expected to remember a part of the history of these interactions and are believed to be most informative about the collision dynamics, hadronization mechanism and may, in principle, carry some relic information about their parent state. The investigation of the multiparticle production process may be interesting and useful for probing the formation of QGP. Since the existence of the phase transition is associated with properties of the nontrivial quantum chromodynamics, the study of quark-hadron phase transition has been a hot point in both particle physics and nuclear physics for more than a decade.

Among the various methods the Levy stable law [3, 4] helps to provide a useful diagnostic tool to detect the existence of possible phase transition in hadronization process. This law is characterized by the Levy stability index μ .This parameter μ , characterizing the width of probability distribution in the elementary partition of random cascade, takes value in the range [0,2] according to the requirement of Levy stability [3, 4]. Within the region of stability $0 \leq \mu \leq 2$, μ has a continuous spectrum. The index μ allows the estimation of the cascading rate [4]. The two bound axes of the Levy index correspond to the degree of fluctuation in the particle production. μ=2 corresponds to the minimum fluctuation from self-similar branching processes. μ = 0 corresponds to the maximum fluctuation that characterizes the interacting system as monofractal [5, 6]. But phase transition cannot be indicated by monofractal behavior alone. According to [4], when μ<1, there is a thermal phase transition (interspersed in the cascading process if μ>0). On the other hand, when μ>1, there is a non-thermal phase transition during the cascading process.

The above discussion about Levy-stability index is for self-similar random cascading process when the Scaled Factorial Moments (SFM) are calculated in the self-similar way, i.e. shrinking the higher dimensional phase space isotropic ally. However, phase space in high-energy multiparticle production is anisotropic as indicated by Van Hove [7]. The fluctuation pattern is also expected to be anisotropic and the scaling behavior should also be different in different directions giving rise to self-affine scaling. In self-affine scenario when the SFMs are calculated, the phase space should be shrunk according to the inherent self-affine parameter – Hurst exponent *H*. The Levy index μ obtained only in this way is meaningful to characterizing the self-affine random cascading process. A very few Levy index analysis in self-affine space have been reported so far [8, 9].

Ginzburg-Landau (GL) formalism [6, 10-12] helps to investigate the existence of second order phase transitions in hadronization process. According to Ginzburg-Landau theory for second order phase transitions, the anomalous fractal dimension (d_q) follows the relation

$$\frac{d_q}{d_2} = (q-1)^{\nu-1}, \tag{1}$$

where scaling exponent ν=1.304 [6, 10-12], a universal quantity that is valid for all systems describable by the GL theory and independent of the underlying dimension or parameters of the model. If the measured value of ν is significantly different from the critical value, then obviously the GL description is inappropriate and second order phase transition can most likely be ruled out. On the other hand, if it is close, then one can expect a second order quark-hadron phase transition.

The universality of ν characterizes quark-hadron phase transitions and can be tested directly by appropriately analyzed data. If a signature of a quark-hadron phase transition depends on the details of the hadron-nucleus collisions, even after they have passed the thresholds for the creation of quark-gluon plasma, such a signature is likely to be sensitive to the theoretical model used. Here, ν is independent of such details and depends only on the validity of the GL description of the phase transition for the present problem.

Pions (shower particles) are believed to be most informative in finding the proper dynamics of the multiparticle production process. But the medium energy (30-400) MeV knocked out protons, which manifest themselves as grey particles in nuclear emulsion, may also play an important role in this regard. It is generally believed that grey particles are supposed to carry relevant information about the hadronization mechanism, since the time scale of emission of these particles is of the same order as that of the produced shower particles and hence are expected to remember a part of the history of these reactions. These target protons are the low energy part of intra-nuclear cascade formed in high-energy interactions.

It is interesting to note that the number of grey particles, n_g, could serve as a measure for the number of collisions in nuclei and, only for qualitative discussion, the mean number of collisions $\acute{v}$ is likely to be a monotonically rising function of n_g eventually saturating at high n_g. In a more general sense, n_g together with the number of pions can be interpreted as a measure of violence of the target fragmentation. It would be no doubt interesting to discuss the behaviour of pions as a function of n_g, which is taken as a number of collisions or, more generally, as a measure of the violence of target fragmentation [13, 14]. To get more information about the inner dynamics of the particle production in high-energy interactions, the phase transition and its dependence on target excitation has to be studied thoroughly using the available tools. To do this, we have divided the data for produced pions for (π^- - AgBr) interactions at 350 GeV into three sets depending upon the number of grey tracks (n_g). The different data sets correspond to different degrees of target excitation.

The aim of the present paper is to perform Levy stability analysis of the produced pions in two dimensional (η - ϕ) phase space under self-affine scenario imposing special emphasis on phase transition study. Levy stable law has been used to determine the value of μ for different target excitations (different values of n_g) in (π^- - AgBr) interactions at 350 GeV/c to asses the dependence of the quark-hadron phase transition on target excitation. Finally using the GL theory we have determined the value of ν for the same interaction to search for the second order quark-hadron phase transition.

Experimental details

We study the hadron-nucleus interaction data of π^- AgBr at 350 GeV/c . A stack of G5 nuclear emulsion plate was exposed horizontally to a π^- - AgBr beam at CERN with 350 GeV/c .

The nuclear emulsion covers 4π geometry and provides very good accuracy, even less than 0.1 mrad, in angle measurements due to high spatial resolution and thus is suitable as a detector for the study of fluctuations in fine resolution intervals of the phase space. The emulsion plates were area scanned with a Leitz Metalloplan Microscope fitted with a semiautomatic scanning device, having a resolution along the X and Y axes of 1 μm while that along the Z axis is 0.5 μm. A sample of 569 events of (π^- - AgBr) at 350 GeV/c was chosen, following the usual emulsion methodology for selection criteria of the events.

The events were chosen according to the following criteria:

i) The incident beam track should not exceed 3^o from the main beam direction in the pellicle. This is done to ensure that we have taken the real projectile beam.

ii) Events showing interactions within 20 μm from the top and bottom surface of the pellicle were rejected. This is done to reduce the loss of tracks as well as to reduce the error in angle measurement.

iii) The incident particle tracks, which induced interactions, were followed in the backward direction to ensure that they indeed were projectile beam starting from the beginning of the pellicle.

The emission angle (θ) and azimuthal angle (ϕ) are measured for each tracks by taking readings of the coordinates of the interaction point (x_0, y_0, z_0), coordinates (x_i, y_i, z_i) at any point on the linear portion each secondary track and coordinate (x_1, y_1, z_1) of a point on the incident beam.

According to nuclear emulsion terminology [15], the particles emitted in high-energy interactions are classified as:

- Black particles: They are target fragments with ionization greater than or equal to $10I_0$, I_0 being the minimum ionization of a singly charged particle. Their ranges are less than 3 mm. Their velocity is less than $0.3c$ and their energy is less than 30 MeV, where c is the velocity of light in free space.

- Grey particles: They are mainly fast target recoil protons with energy up to 400 MeV. The ionization power of gray particles lies between $1.4I_0$ to $10I_0$. Their ranges are greater than 3 mm and they have velocities between $0.3c$ to $0.7c$.

- Shower particles: They are mainly pions with ionization $\leq 1.4I_0$. These particles are generally not confined within the emulsion pellicle.

Methodology

The method of scaled factorial moment is used here to analyse the intermittent type of fluctuations of emitted particles in two-dimensional phase space. Denoting the two-phase space variables as x_1 and x_2, factorial moment of order q may be defined as [16]

$$F_q(\delta x_1, \delta x_2) = \frac{1}{M} \sum_{m}^{M} = 1 \frac{\langle n_m(n_m - 1).......(n_m - q + 1) \rangle}{\langle n_m \rangle^q}, \quad (2)$$

where δx_1 and δx_2 is the size of a two-dimensional cell. The brackets $\langle\ \rangle$ denote the average over the whole ensemble of events. n_m is the multiplicity in the $m^t h$ cell. M is the number of two-dimensional cells into which the considered phase space has been divided.

One has to connect δx_1 , δx_2 and M. As the starting point to solve this problem let us fix a two-dimensional region Δx_1 Δx_2 and divide it in to sub cells of width $\delta x_1 = \Delta x_1 / M_1$ and $\delta x_2 = \Delta x_2 / M_2$. Here M_1 is the number of bins along x_1 direction and M_2 is the number of bins along x_2 direction. Cell size dependence of factorial moment is studied by shrinking the bin widths in both directions. There are two ways of doing it. Widths may be shrinked equally ($M_1 = M_2$) or unequally ($M_1 \neq M_2$) in the two dimensions. The shrinking ratios along x_1 and x_2 directions are characterised by a parameter $H = lnM_1/lnM_2$ where $0 < H \leq 1$ is called the roughness or Hurst exponent [17, 18]. If and only if the shrinking ratios along the two directions satisfy the above relation with a particular H value, the function F_q (δx_1 , δx_2) will have a well defined scaling property. H=1 signifies that the phase space is divided isotropically and consequently fluctuations are self-similar. When H<1 it is clearly understood that the phase spaces along x_1 and x_2 directions are divided anisotropically consequently the fluctuations are self affine in nature.

As noted, the intermittent behavior of the multiplicity distribution manifests itself as a power law dependence of factorial moment on the cell size as cell size approaches zero,

$$\langle F_q \rangle \propto M^{\alpha_q}. \tag{3}$$

The index α_q is obtained from a linear fit of the form

$$ln \langle F_q \rangle = \alpha_q lnM + a, \tag{4}$$

where a is a constant.

According to the predictions of a simple scale – invariant cascade model [3], the higher order scale factorial moments are related to the second order scaled factorial moments by a modified power law

$$F_q \propto F_2^{\beta_q}. \tag{5}$$

which may provide some vital information about the underlying dynamics. It has been found that the slopes of the power law between higher order and second order SFM s are independent of phase space size and phase dimension [6, 10]. In other words the values of β_q summarize the scale invariance property on the global scale.

Now we can define the quantity β_q in terms of the ratio of higher order intermittency exponent to the second order intermittency exponent (or in terms of the ratio of higher order anomalous fractal dimension to the second order anomalous fractal dimension) by the following relation

$$\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{d_q}{d_2}(q-1), \tag{6}$$

β_q is related to Levy index (μ) by the equation

$$\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{q^\mu - q}{2^\mu - 2}. \tag{7}$$

Here, μ, known as Levy index, is considered a measure of degree of multifractality [6]. Within the region of stability $0 \leq \mu \leq 2$, μ has a continuous spectrum.

Note that if μ =2, the Levy distribution will be transformed into Gaussian one. Under this condition one expects minimum fluctuation in the self-similar branching process. On the other hand, for $\mu = 0$, $d_q = d_2$ i.e. d_q does not depend on q, corresponds to monofractality and maximum fluctuation and might, therefore be a signal of QGP second order phase transition. When $\mu > 0$, $d_q \neq d_2$ i.e. d_q depends on q, the condition for multifractality is satisfied.

According to GL theory for second order phase transition the anomalous fractal dimension follows the relation

$$\frac{d_q}{d_2} = (q-1)^{\mu-1}. \tag{8}$$

Or in terms of β_q the scaling behavior is represented by the following relation

$$\beta_q = (q-1)^\mu, \tag{9}$$

with $\nu = 1.304$ as the critical exponent.

Results and Discussion

In order to reduce the effect of non-flat average distribution, the cumulative variables X_η and X_ϕ are used instead of η and ϕ [19, 20]. The corresponding region of investigation then becomes (0-1). The new "cumulative" variable X_z is related to the original single-particle density distribution $\rho(z)$ as,

$$X_z = \int_{z_{min}}^{z} \rho(z')\delta z' / \int_{z_{min}}^{z_{max}} \rho(z')\delta z', \tag{10}$$

where z_{min} and z_{max} are the two extreme points of the distribution. In the $X_\eta - X_\phi$ space we divided the region [0, 1] into M_η and M_ϕ bins respectively. The partitioning was taken as $M_\eta = M_\phi^H$. We choose the partition number along ϕ direction as M_ϕ=2, 3, ..., 20. The ($X_\eta - X_\phi$) space is divided into $M = M_\eta \times M_\phi$ cells and calculation is done in each bin independently.

To analyze the anisotropic nature of pions in the ($X_\eta - X_\phi$) phase space factorial moment of different orders for different Hurst exponents starting from 0.3 to 0.7 in steps of 0.1 and for $H = 1$ are calculated. We have studied the variation of average factorial moment $\langle F_q \rangle$ against the number of the two dimensional cells M in a log-log plot for different orders (q=2, 3, 4 and 5) and for the considered H values. In order to find the partitioning condition at which the scaling behavior is best revealed, we have performed the linear best fits. From the linear best fits intermittency exponents (α_q) are extracted. $\chi^2/d.o.f.$ values are calculated for each linear fits. We have also estimated the confidence level of fittings from the χ^2 values. The minimum value of χ^2 per degree of freedom indicates the best linear behavior. For pions the best linear fit occurs at H=0.3 which shows that the anisotropic behavior is best revealed at H=0.3. The values of χ^2 per degree of freedom and the confidence level of fittings are tabulated in Table 1 for H=0.3. From the table it is seen that for H=0.3 the confidence level of fittings are very good. For this H value the plot of $ln \langle F_q \rangle$ as a function of lnM is shown in Figure 1(a). To compare the self- affine behaviour with the self similar one the variation of $ln \langle F_q \rangle$ against ln$M$ corresponding to H=1 is shown in Figure 1(b) and the corresponding results are shown in Table 1. χ^2 per degree of freedom values and confidence level of fittings at H=0.3 are better than the corresponding values obtained at H=1. So the dynamical fluctuation pattern of shower particles in π^- - AgBr interaction at 350 GeV/c is not self-similar but self-affine in nature.

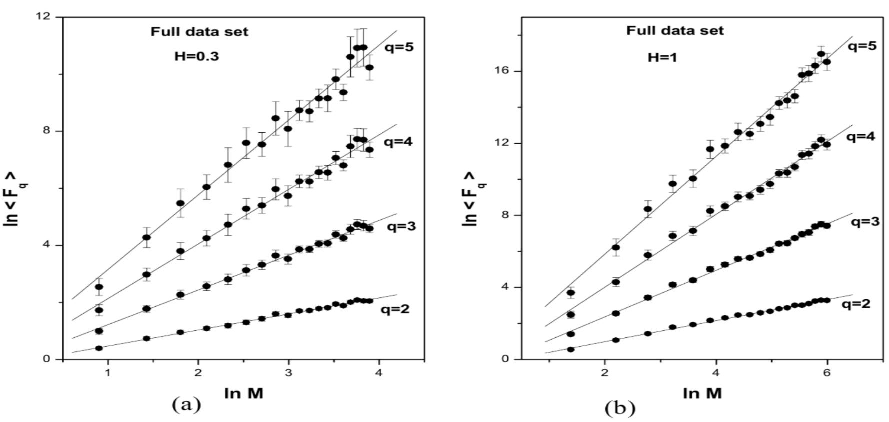

Figure 1. Variation of $ln \langle F_q \rangle$ as a function of on lnM for full data set at (a): H = 0.3, (b): H = 1.

Using the values of α_q, we have calculated β_q using Equation (6). These values are given in Table 2. The β_q versus q graph is shown in Figure 2. It is observed that the parameter β_q increases with increasing order of moments. This indicates the fact that charged particle density distribution has multifractal structure. Therefore, we can say that hadrons in the final state are produced as a result of a self-similar cascade mechanism [3, 16].

Table 1.
Values of intermittency exponent α_q, $\chi^2/d.o.f.$ and confidence level of fittings at H=0.3 and H=1 for full data set

H	q	α_q	$\chi^2/d.o.f.$	Confidence level of fittings
0.3	2	0.56 ± 0.01	0.38	98.90 %
	3	1.22 ± 0.03	0.49	95.93 %
	4	1.92 ± 0.06	0.58	91.30 %
	5	2.63 ± 0.10	0.70	80.95 %
1	2	0.59 ± 0.01	0.74	76.62 %

Then Levy stability index μ is calculated using Equation (7) and tabulated in Table 2. Here the Levy index obtained for the $(\eta - \phi)$ space is $\mu = 0.468 \pm 0.005$, which is within the permissible limit $0 \leq \mu \leq 2$. Here $\mu < 1$ would have indicated a thermal phase transition of second order.

Table 2.
Values of different parameters (β_q, μ and ν) for full data set

H	q	β_q	μ	ν
0.3	2	0.56 ± 1	0.468 ± 0.005	1.110 ± 0.002
	3	1.22 ± 2.16		
	4	1.92 ± 3.39		
	5	2.63 ± 4.65		

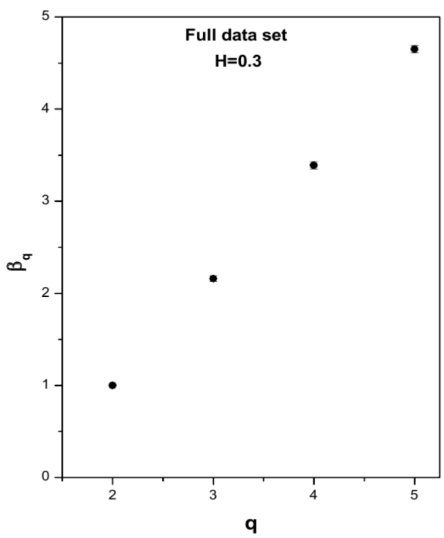

Figure 2. Variation of β_q with q for full data set for self-affine H, 1 case.

On the basis of the Ginzburg – Landau (GL) theory it has been found that the scaling exponent $\nu = (1.110 \pm 0.002)$. This value of ν (considering the errors) is significantly different from the critical value, 1.304. This makes the GL description inappropriate, and no second order QGP phase transition takes place in the hadronization process.

For studying target excitation dependence we have divided the data set for pions into three sets, $0 \leq n_g \leq 2$, $3 \leq n_g \leq 5$, $6 \leq n_g \leq 13$, depending on the number of grey tracks (n_g).The sets correspond to different degrees of target excitation. The division is made in such a way that each set contains reasonably equal number of events. The self – affine analysis is repeated for the three data sets. The fluctuation pattern is self-affine in nature in all the three sets of n_g . $ln \langle F_q \rangle$ vs. lnM graphs for all n_g intervals are shown in Figure 3 and corresponding results are tabulated in Table 3. A low value of H suggests that anisotropy is strong for $0 \leq n_g \leq 2$ and $6 \leq n_g \leq 13$ data sets.

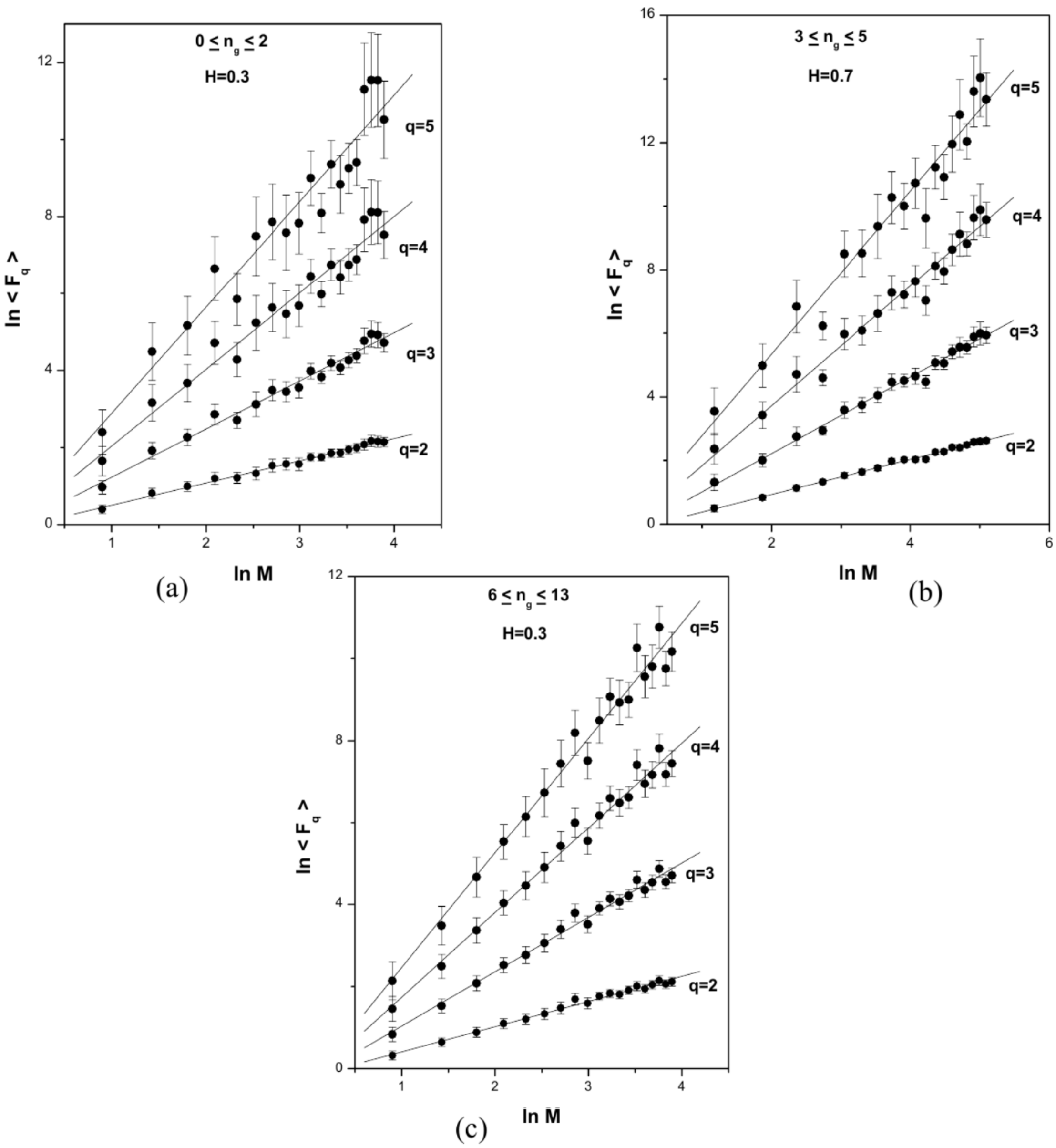

Figure 3. Variation of $ln \langle F_q \rangle$ as a function of on lnM for self-affine H, 1 case for different intervals (a): $0 \leq n_g \leq 2$, (b): $3 \leq n_g \leq 5$, (c): $6 \leq n_g \leq 13$.

Table 3.
Values of intermittency exponent α_q , $\chi^2/d.o.f.$ and confidence level of fittings for different n_g intervals (for self-affine H, 1 case)

n_g	H	q	α_q	$\chi^2/d.o.f.$	Confidence level of fittings
$0 \leq n_g \leq 2$	0.3	2	0.58 ± 0.01	0.13	Almost 100 %
		3	1.25 ± 0.05	0.51	94.89 %
		4	1.99 ± 0.10	0.54	93.67 %
		5	2.76 ± 0.18	0.70	80.67 %
$3 \leq n_g \leq 5$	0.7	2	0.55 ± 0.01	0.33	Almost 100 %
		3	1.20 ± 0.03	0.49	95.82 %
		4	1.88 ± 0.07	0.67	84.04 %
		5	2.56 ± 0.13	0.77	72.95 %
$6 \leq n_g \leq 13$	0.3	2	0.61 ± 0.02	0.24	Almost 100 %
		3	1.32 ± 0.04	0.56	92.07 %
		4	2.07 ± 0.07	0.57	91.55 %
		5	2.80 ± 0.10	0.59	90.61 %

In the self-affine space the Levy index analysis is performed for the three target excitation data sets. The β_q versus q graphs for the three data sets are shown in Figure 4. It is observed that the parameter β_q increases with increasing order of moments revealing multifractal pattern of produced pions in different n_g intervals. The errors shown in the figures are standard errors. The values of μ are calculated following the same procedure as in the previous cases and listed in Table 4. We get μ<1 for three target excitation data sets indicating a second order thermal phase transition (interspersed in the cascading process) with a largelatent heat, and thus may serve as a possible indication of QGP being formed.

Table 4.
Values of different parameters (β_q, μ and ν) for different n_g intervals (for self-affine H, 1 case)

n_g	H	q	β_q	μ	ν
$0 \leq n_g \leq 2$	0.3	2	1	0.542±0.002	1.131±0.004
		3	2.17		
		4	3.46		
		5	4.79		
$3 \leq n_g \leq 5$	0.7	2	1	0.478±0.002	1.112±0.004
		3	2.18		
		4	3.42		
		5	4.66		
$6 \leq n_g \leq 13$	0.3	2	1	0.425±0.012	1.098±0.007
		3	2.16		
		4	3.37		
		5	4.57		

Moreover, the values of Levy indices (μ) vary consistently with degrees of target excitation.

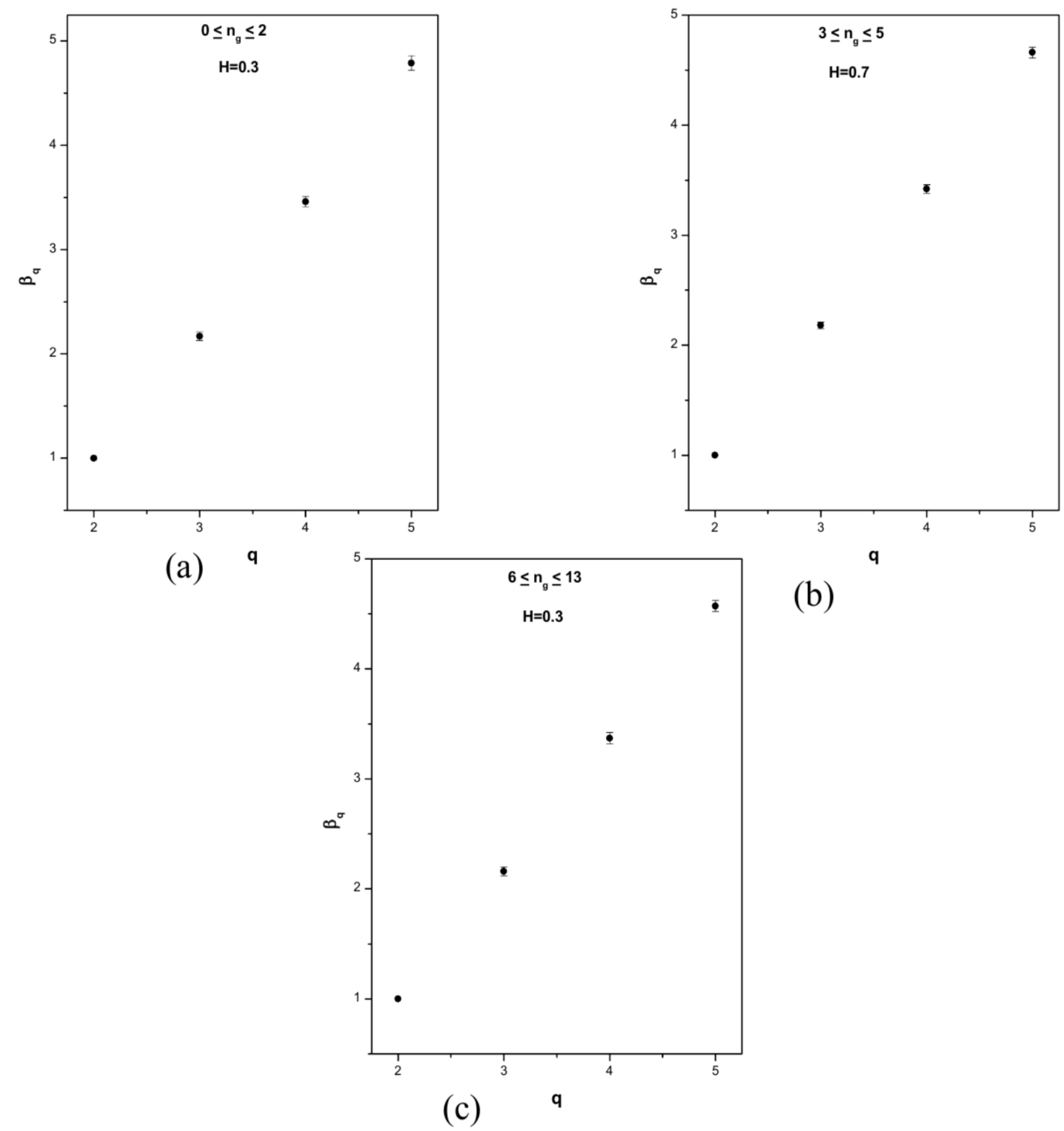

Figure 4. Variation of β_q with q for self-affine H, 1 case for different n_g intervals (a): $0 \leq n_g \leq 2$, (b): $3 \leq n_g \leq 5$, (c): $6 \leq n_g \leq 13$.

Again according to the GL theory the values of ν for three n_g intervals are calculated and are listed in Table 4. From the table it is observed that the values of ν are significantly different from the critical value of ν making the GL description inappropriate and second order phase transition can most likely be ruled out.

Conclusions

Quark-hadron phase transition and its dependence on target excitation have been studied for pions in self-affine $\eta-\phi$ phase space. The following interesting features are revealed from the present investigation:

1. The parameter β_q increases with increasing order of moments q, which indicates that self-similar cascading to be the mechanism responsible for multiparticle production. From our analysis we find that the particle density distribution possesses multifractal structure and the degree of multifractality is different for different target excitations.

2. The values of the Levy stability index μ obtained in our study are consistent with the Levy stable region $0 \leq \mu \leq 2$.

3. We get μ<1 for full data set and as well as for three target excitation data sets indicating a second order thermal phase transition and thus may serve as a possible indication of QGP being formed.

4. The values of Levy indices (μ) vary consistently with degrees of target excitation.

5. From the values of the critical exponent ν in our analysis, no evidence for the existence of second order phase transition has been found according to the GL theory.

6. The values of critical exponents (ν) vary consistently with degrees of target excitation.

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